Chapter 4

Binomial coefficient

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Theorem 4.2 Summation Formulas

Let n ε Z with n ≥ 1, then

(a) $\sum_{i=1}^{n} 1 = n.$

(b) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}.$

(c) $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}.$

(d) $\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$

Fundamental Theorem of Arithmetic

Every integer greater than 1 has a unique standard factorization.

$n = p_1^{e_1} \cdot p_2^{e_2} \cdots\cdots p_n^{e_n}$, where *m* is a positive integer; $p_1, \ldots, p_n$ are primes, and $e_1, \ldots, e_n$ are positive integers.

Binomial Theorem

Let *a, b* ε Z^+. Then,

$$(a+b)^n = \sum_{i=0}^{n} \binom{n}{i} a^{n-i} b^i$$

Useful Identities

$$\forall n \geq 0, \sum_{i=0}^{n} \binom{n}{i} = 2^n, \quad \sum_{i=0}^{n} \binom{n}{i} 2^i = 3^n$$

Pascal's Triangle, binomial coefficients, and the sum of each row.