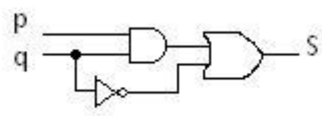
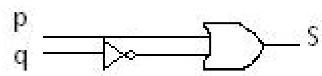
| | | | |
|---|---|---|---|
| median | 73 | A | 4 |
| avg | 76 | B | 4 |
| max | 97 | C | 6 |
| count | 21 | D | 3 |

Discrete Structures Test #1 S 2009

1) Determine a **simplified** circuit for S = ($p \wedge q$) v ¬$q$. *Show your reduction work and draw the simplified circuit.*




¬$q$ v ($q \wedge p$) *associative law*
(¬$q$ v $q$) ^ (¬$q$ v $p$) *distributive law*
t ^ (¬$q$ v $p$) *negation law*
¬$q$ v $p$ *identity law*




2) Verify the logical equivalence $p \wedge q \rightarrow r \equiv \neg p \vee (q \rightarrow r)$ by using Theorem 1.1, 1.2 and the alternate representation of the conditional $p \rightarrow q \equiv \neg p \vee q$.

$p \wedge q \rightarrow r \equiv \neg(p \wedge q) \vee r$ *Alternate representation of conditional*
$\equiv(\neg p \vee \neg q) \vee r$ *DeMorgan's*
$\equiv \neg p \vee (\neg q \vee r)$ *Associativity*
$\equiv \neg p \vee (q \rightarrow r)$ *Alternate representation of conditional*

3) Reduce $A \cup (B \cap C^c)^c \cup C$

$A \cup (B^c \cup C) \cup C$ *DeMorgan's*
$A \cup B^c \cup (C \cup C)$ *Associative*
$A \cup B^c \cup C$ *Idempotent*

4) *Symmetric difference* between two sets A and B is: $A \Delta B = A \backslash B \cup B \backslash A$. Verify the identity stated below.

$(A \cap B \cap C^c) \cup (A \cap B^c \cap C) = A \cap (B \Delta C)$
$= (A \cap (B \cap C^c)) \cup (A \cap (B^c \cap C))$ *associative*
$= A \cap ((B \cap C^c) \cup (B^c \cap C))$ *distributive*
$= A \cap ((B \setminus C) \cup (C \setminus B))$ *set difference*
$= A \cap (B \Delta C)$ *symmetric difference*

5) T/F

a) $\{4\} \subseteq \{\{2\}, \{4\}, \{6\}\}$ F
b) $4 \in \{n \mid n = 2k, \forall k \in Z\}$ T
c) $4 \in \{\{2\}, \{4\}, \{6\}\}$ F
d) $\{4\} \in P(\{1, 2, 3, 4\})$ T
e) $\{4\} \subseteq \{1, 2, 3, 4\}$ T
f) $\{2, 4, 6\} \subset \{2, 4, 6\}$ F

6) Write out the symbolic representation of the Quotient-Remainder Theorem. Given any integer $n$ and any positive integer $d$, there exist integers $q$ and $r$ such that:

$n = dq + r, 0 \leq r < d.$

7) Use Euclid's Division Algorithm to find gcd(330, 156). *Show your work.*

330 = 156*2 + 18 gcd(n, d) = gcd (d, r)
156 = 18*8 + 12
18 = 12*1 + **6**
12 = 6*2 + 0 gcd(330, 156) = **6** *the last non-zero remainder.*

8) Let $g$ be the function given by

$$g(n) = \begin{cases} \frac{n}{2} & if \ \ n \ is \ even \\ \frac{3n+1}{2} & if \ \ n \ is \ odd \end{cases}$$

Show that for positive integer $n$ = 10, starting from $n$ and iterating $g$, the function value eventually returns to 1.

10 → 5 → 8 →4 →2 → 1

9) Given the Java integer: 1100 0010 0000 0011 1000 0001 1001 0100
Java uses 2's complement representation and a 32 bit word size.

a) Hexadecimal Representation: C2038194
b) Decimal Value: -1039957612

-0011 1101 1111 1100 0111 1110 0110 1100
-3 D F C 7 E 6 C
$-3*16^7+D(13)*16^6+F(15)*16^5+C(12)*16^4+7*16^3+E(14)*16^2+6*16+C(12)$

10) Convert 188.09375 to octal and to binary:

188.09375 = $274.06_8$ = $010\ 111\ 100.000\ 110_2$

8 |188
8 |23 r 4.
8 |2 r 7
0 r 2

.09375
x8
0.75
x8
6.0 $2*8^2 + 7*8 + 4*8^0 + 0*8^{-1} + 6 * 8^{-2} = 188.09375$ √

11) Consider the Java bytes A = 1010 1010, B = 0101 0101. Convert their sum, A + B, to decimal:

1010 1010
0101 0101
1111 1111
-0000 0001
$-1_{10}$

12) Write the negation of the given statement symbolically and determine whether the original or the negation is true.
There is a natural number *n* such that, for every real number *x*, $x^n$ is nonnegative.
Original: true
$\exists\, n \in N$ such that $\forall x \in R, x^n \geq 0$.
Negation: false
$\forall\, n \in N, \exists\, x \in R$ such that $x^n < 0$.

13) Prove: The product of an even integer and an odd integer is even.

Proof: Suppose *m* is an even integer and *n* is an odd integer. *[We must show m * n is even.]*
$m = 2\,k,\ k \,\varepsilon\, Z$
$n = 2\,l + 1,\ l \,\varepsilon\, Z$
$m * n = 2k\,(2\,l + 1)$
$= 2\,k\,2\,l + 2\,k$
$= 2\,(2\,k\,l + k)$ □

14) Complete the values table for the following algorithm with a[] = {1, 5, 6}, s = 8, and length = 3.

```
int decimalValue (int a[], int s, int length) {
   int i, n = 0;
   for (i = 0; i < length; i++) {
      n += a[length - 1 - i] * pow(s,i);
   }
   return n;
}
```

| i | a[length - 1 - i] | pow(s, i) | a[length - 1 - i] * pow(s, i) | n |
|---|---|---|---|---|
| 0 | 6 | 1 | 6 | 6 |
| 1 | 5 | 8 | 40 | 46 |
| 2 | 1 | 64 | 64 | 110 |
| 3 | | | | |
| 4 | | | | |