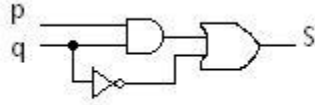


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Discrete Structures Test #1 S 2009

- 1) Determine a **simplified** circuit for  $S = (p \wedge q) \vee \neg q$ . Show your reduction work and draw the simplified circuit.



- 2) Verify the logical equivalence  $p \wedge q \rightarrow r \equiv \neg p \vee (q \rightarrow r)$  by using Theorem 1.1, 1.2 and the alternate representation of the conditional  $p \rightarrow q \equiv \neg p \vee q$ .

- 3) Reduce  $A \cup (B \cap C^c)^c \cup C$  :

- 4) *Symmetric difference* between two sets A and B is:  $A \Delta B = A \setminus B \cup B \setminus A$ . Verify the identity stated below.

$$(A \cap B \cap C^c) \cup (A \cap B^c \cap C) = A \cap (B \Delta C)$$

- 5) T/F

- a)  $\{4\} \subseteq \{\{2\}, \{4\}, \{6\}\}$
- b)  $4 \in \{n \mid n = 2k, \forall k \in \mathbb{Z}\}$
- c)  $4 \in \{\{2\}, \{4\}, \{6\}\}$
- d)  $\{4\} \in \mathcal{P}(\{1, 2, 3, 4\})$
- e)  $\{4\} \subseteq \{1, 2, 3, 4\}$
- f)  $\{2, 4, 6\} \subset \{2, 4, 6\}$

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6) Write out the symbolic representation of the Quotient-Remainder Theorem. Given any integer  $n$  and any positive integer  $d$ , there exist integers  $q$  and  $r$  such that:

7) Use Euclid's Division Algorithm to find  $\text{gcd}(330, 156)$ . *Show your work.*

8) Let  $g$  be the function given by

$$g(n) = \begin{cases} \frac{n}{2} & \text{if } n \text{ is even} \\ \frac{3n+1}{2} & \text{if } n \text{ is odd} \end{cases}$$

Show that for positive integer  $n = 10$ , starting from  $n$  and iterating  $g$ , the function value eventually returns to 1.

9) Given the Java integer: 1100 0010 0000 0011 1000 0001 1001 0100

Java uses 2's complement representation and a 32 bit word size.

a) Hexadecimal Representation:

b) Decimal Value:

10) Convert 188.09375 to octal and to binary:

11) Consider the Java bytes  $A = 1010\ 1010$ ,  $B = 0101\ 0101$ . Convert their sum,  $A + B$ , to decimal:

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12) Write the **negation** of the given statement symbolically and **determine** whether the original or the negation is **true**.

There is a natural number  $n$  such that, for every real number  $x$ ,  $x^n$  is nonnegative.

13) Prove: The product of an even integer and an odd integer is even.

14) Complete the values table for the following algorithm with  $a[] = \{1, 5, 6\}$ ,  $s = 8$ , and  $\text{length} = 3$ .

```
int decimalValue (int a[], int s, int length) {  
    int i, n = 0;  
    for (i = 0; i < length; i++) {  
        n += a[length - 1 - i] * pow(s, i);  
    }  
    return n;  
}
```

i	a[length - 1 - i]	pow(s, i)	a[length - 1 - i] * pow(s, i)	n
0				
1				
2				
3				
4				