avg	87		A	11
max	102.5 (q11 up to 4 pts bonus)		B	3
count	21		C	5
			D	2

Discrete Structures Test #2 S 2009

1) Draw a line to connect each equivalent statement:

a) $(A \cap B) \cup \varnothing$	$A \cup A^c$
b) $A \cap A^c$	$A - B^c$
c) $A - B$	$\varnothing$
d) U	$A \cap B^c$

2) Reduce $A \cup (B \cap C^c)^c \cup C$:

$A \cup (B^c \cup C) \cup C$ by DeMorgan

$A \cup B^c \cup (C \cup C)$ by associativity

$A \cup B^c \cup C$ by Idempotent

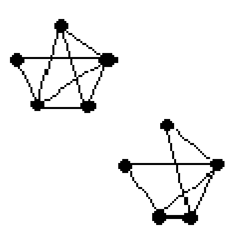
3) Markov chains are employed in algorithmic music composition. In a first-order chain, the states of the system become note or pitch values, completing a transition probability matrix (see below). An algorithm is constructed to produce and output note values based on the transition matrix weightings. Determine the probability if the present note is A, followed by another note, then the next note is Eb. (Hint:a cell in Note^2.) $\text{Note}^2[A, Eb] = .45$

Note	A	C#	Eb
A	.1	.6	.3
C#	.25	.05	.7
Eb	.7	.3	0

4) Determine $\sum_{i=-3}^{4} a_i 2^i$, $a_{-3} = 1, a_{-2} = 0, a_{-1} = 0, a_0 = 0, a_1 = 0, a_2 = 1, a_3 = 1, a_4 = 1$.

28.125

5) Are these two graphs isomorphic? If so label their vertices so that the isomorphism is apparent. If not, list an isomorphic invariant that is violated. NO, score sequence, # edges, # 3-cycles, ...



avg 87
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count 21

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6) The relation R = {(a, a), (b, b), (b,c), (b,d), (c,b), (c,c), (c,d), (d, b), (d, c), (d, d)} is an equivalence relation on the set L = {a, b, c, d}.
   a) Draw directed graph for the relation R.

   b) If R is reflexive, symmetric and transitive, find the distinct equivalence classes of R.
   There are two distinct equivalence classes: [a], [b]=[c]=[d].

   c) Create an adjacency matrix for R on L x L.
   1 0 0 0
   0 1 1 1
   0 1 1 1
   0 1 1 1
   d) Find R^2
   1 0 0 0
   0 3 3 3
   0 3 3 3
   0 3 3 3

7) List the first three terms of the sequence $c_i = (-1)^{i+1} / 2^{i+1}$, for all integers $i \geq 0$:
   -1/2, 1/4, -1/8

```
public static void main(String[] args) {
      int[] f = {1,6,2,-9,6};
      int[] g = new int[f.length];
      int a = -2, i;
      g[0] = f[0];
      for(i = 0; i < f.length; i++) {
            System.out.print(" f["+ i + "] = " + f[i]+ (i==f.length-1?
            ".\n":", "));
      }
      for(i = 1; i < f.length; i++) {
            g[i] = a*g[i-1] + f[i];
      }
      System.out.println((g[i-1]==0 ? " Found root: x = " + a :
                        " remainder: " + g[i-1] + "/(x - " + a + ")"));
}
//output:
f[0] = 1,  f[1] = 6,  f[2] = 2,  f[3] = -9,  f[4] = 6.
 Found root: x = -2
```

8) What are the final values of g?
   {1, 4, -6, 3, 0}

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9) Consider the Java bytes A = 1010 1010, B = 0101 0101. Convert their sum, A + B, to decimal:
1111 1111 = -1 (2's complement rules apply)

10) Complete this Proof by mathematical induction 1) basis step 2)inductive step.

P(*n*): 1 + 4 + 7 + 10 + … + (3*n* – 2) = (*n*/2)(3*n* – 1), for all *n* greater than or equal to 1.

Proof (MI) Basis Step:

$$P(n): \sum_{i=1}^{n}(3i-2) = \frac{n}{2}(3n-1), \quad \forall n \in Z, n \geq 1$$

$$P(1): \sum_{i=1}^{1}(3i-2) = \frac{1}{2}(3(1)-1)$$

$$3(1)-2 = \frac{1}{2}(2)$$

$$1 = 1$$

Inductive Step:

$$\text{Let } P(k): \sum_{i=1}^{k}(3i-2) = \frac{k}{2}(3k-1), \quad \text{for some integer } k \geq 1.$$

We must show $P(k+1)$ is true.

$$P(k+1): \sum_{i=1}^{k}(3i-2) + (3(k+1)-2) = \frac{k+1}{2}(3(k+1)-1).$$

$$\text{But } \sum_{i=1}^{k}(3i-2) = \frac{k}{2}(3k-1) \text{ by inductive hypothesis .}$$

$$\text{So } P(k+1) \text{ is then : } \frac{k}{2}(3k-1) + (3(k+1)-2) = \frac{k+1}{2}(3(k+1)-1)$$

$$= \frac{3k^2-k}{2} + (3k+1) = \frac{k+1}{2}(3k+2)$$

$$= \frac{3k^2-k}{2} + \frac{2}{2}(3k+1) = \frac{k+1}{2}(3k+2)$$

$$= \frac{3k^2-k}{2} + \frac{2}{2}(3k+1) = \frac{k+1}{2}(3k+2)$$

and we have

$$\frac{3k^2+5k+2}{2} = \frac{3k^2+5k+2}{2}.$$

avg 87

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11) Given the set H = {1, 2, 3} Plus one point for each correct answer (max 4)

a) Which are equivalence relations? Reflexive, Symmetric and Transitive → only **T**

b) Which are partial order relations? Reflexive, Anti-symmetric and Transitive → **none**

c) Which relations have sources? In-degree = 0 → **none**
List the sources:

d) Which relations have sinks? Out-degree = 0 → one vertex in R
List the sinks: **R3**

R	**1**	**2**	**3**
1	0	1	1
2	1	1	0
3	0	0	0

S	**1**	**2**	**3**
1	1	1	1
2	0	1	0
3	1	1	1

T	**1**	**2**	**3**
1	1	0	1
2	0	1	0
3	1	0	1

V	**1**	**2**	**3**
1	0	0	1
2	0	1	0
3	1	0	0

W	**1**	**2**	**3**
1	0	1	0
2	1	0	1
3	0	1	0

R, V, W are not reflexive

S is neither symmetric nor anti-symmetric