Math 112 **Worksheet for sections 8.1 and 8.2**

$y = \sin^{-1} x$ *means* $x = \sin y$ *where* $-1 \le x \le 1$ *and* $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ *notice: quadrant I and IV*

$y = \tan^{-1} x$ *means* $x = \tan y$ *where* $-\infty \le x \le \infty$ *and* $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ *notice: quadrant I and IV*

$y = \cos^{-1} x$ *means* $x = \cos y$ *where* $-1 \le x \le 1$ *and* $0 \le y \le \pi$ *notice: quadrant I and II*

1. Evaluate without a calculator giving exact values, since these angles are "special angles". Draw a sketch of the angle and label the point on a unit circle to illustrate each one.

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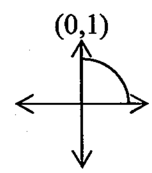
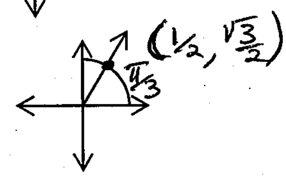
|---|
| For example: *If* $\sin\theta = x$, *then* $\sin^{-1} x = \theta$. *Thus if* $\sin\frac{\pi}{6} = \frac{1}{2}$, *then* $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$. |

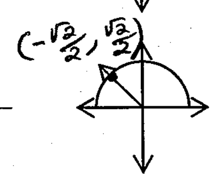
a. $\sin\left(\frac{\pi}{2}\right) =$ $\underline{1}$ so $\sin^{-1}(1) =$ $\underline{\pi/2}$




b. $\tan\left(\frac{\pi}{3}\right) =$ $\underline{\sqrt{3}}$ so $\tan^{-1}\left(\sqrt{3}\right) =$ $\underline{\pi/3}$

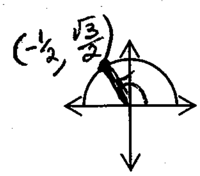



c. $\cos\left(\frac{3\pi}{4}\right) =$ $\underline{-\frac{\sqrt{2}}{2}}$ so $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) =$ $\underline{\frac{3\pi}{4}}$




d. $\tan\left[\cos^{-1}\left(-\frac{1}{2}\right)\right] = \tan\left[\frac{2\pi}{3}\right] =$ $\underline{-\sqrt{3}}$

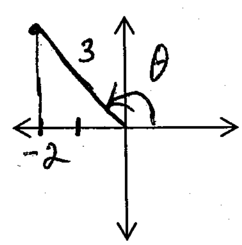
(angle whose cosine is $-\frac{1}{2}$)




2. Evaluate without a calculator giving an exact value. Draw and label a right triangle on these axes to illustrate how to solve this problem.

$\cot\left[\cos^{-1}\left(-\frac{2}{3}\right)\right] =$ $\underline{\cancel{-\frac{2}{\sqrt{5}}}}$ or $-\frac{2\sqrt{5}}{5}$

$\cot(\theta) = \dfrac{\text{adjacent side}}{\text{opposite side}}$




$(-2)^2 + b^2 = 3^2$

$b^2 = 9 - 4$

$b^2 = 5$

$b = \sqrt{5}$

6. Use a calculator (set in radian mode) to find the value of each expression. Show how you are calculating each of these and round each answer to two decimal places.

a. $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) = -.5235987\ldots$

$\approx -.52$

Angle in quadrant 4.

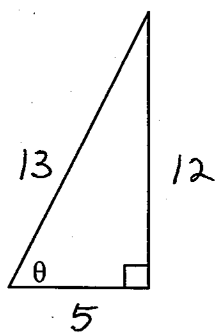
b. $\csc^{-1}\left(\frac{4}{3}\right) = \sin^{-1}\left(\frac{3}{4}\right)$

$= .84806\ldots$

$\approx .85$

**Finding the exact value of expressions involving inverse trig functions:**

7. Find other trig functions of the angle θ in the right triangle show below, if $\theta = \sin^{-1}\left(\frac{12}{13}\right)$.



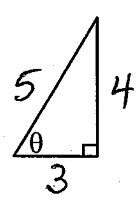

Recall that if $\theta = \sin^{-1}\left(\frac{12}{13}\right)$, then $\sin\theta = \frac{12}{13}$.

Find the exact value of $\tan\left(\sin^{-1}\left(\frac{12}{13}\right)\right) = \frac{12}{5}$

Find the exact value of $\cos\left(\sin^{-1}\left(\frac{12}{13}\right)\right) = \frac{5}{13}$

Using your calculator, find the approximate value of θ. $\theta = \sin^{-1}\left(\frac{12}{13}\right) \approx 1.176$ radians

8. Find other trig functions of the angle θ in the right triangle show below, if $\theta = \cos^{-1}\left(\frac{3}{5}\right)$.




Recall that if $\theta = \cos^{-1}\left(\frac{3}{5}\right)$, then $\cos\theta = \frac{3}{5}$.

Find the exact value of $\sin\left(\cos^{-1}\left(\frac{3}{5}\right)\right) = \sin(\theta) = \frac{4}{5}$

Find the exact value of $\tan\left(\cos^{-1}\left(\frac{3}{5}\right)\right) = \tan(\theta) = \frac{4}{3}$

Using your calculator, find the approximate value of θ.

$\theta = \cos^{-1}\left(\frac{3}{5}\right) = .927295218\ldots$

$\approx .93$ radians

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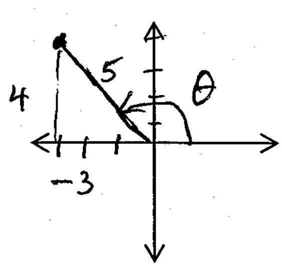
9. Find the exact value of $\sin\left(\cos^{-1}\left(-\frac{3}{5}\right)\right) = \boxed{\frac{4}{5}}$

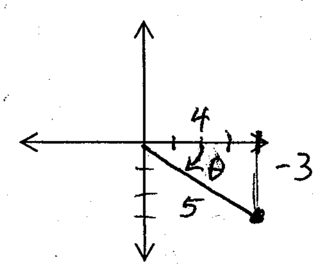



Thus, $\cos\theta = -\frac{3}{5}$

so

$\sin\theta = \frac{4}{5}$

10. Find the exact value of $\cos\left(\sin^{-1}\left(-\frac{3}{5}\right)\right) = \boxed{\frac{4}{5}}$




Thus $\sin\theta = -\frac{3}{5}$

so $\cos\theta = \frac{4}{5}$