5.5 – 5.6 Polynomial Functions

I. Rational Roots Theorem

Let f(x) be a polynomial function of degree 1 or higher of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$

where each coefficient is an integer. If p/q , in lowest terms is a rational zero of f, then **p** must be a factor of a_0 and **q** must be a factor of a_n .

Possible Rational Roots (PPR): $\dfrac{\textbf{Divisors of the constant}}{\textbf{Divisors of the leading coefficient}}$

Think about this example: $f(x) = 2x^2 - x - 15$. Factor and find the zeros. See how they illustrate the theorem above.

Steps for Finding Zeros of a Polynomial and Graphing

Step 1: Use the **degree** of the polynomial to determine the **maximum number of Zeros** and **turning points**.

Step 2: Use the **leading coefficient** to determine the **end behavior** of the polynomial

Step 3: If the polynomial has *integer* coefficients, use the **Rational Zeros Theorem** to **identify those rational numbers** that are potential zeros. (*See if there are any common factors first and factor out the common factor first.)*

Step 4: Use your **calculator table to test PPR** (Possible rational roots/zeros) to determine the actual rational zeros of the polynomial.

Step 5: Use **synthetic division** to help you to **write the polynomial in factored form**. This will allow you to find the zeros that are not rational. (irrational, Imaginary, or complex numbers.)
A review of synthetic division can be found on page 57 of your textbook.

Step 6: Graph the polynomial labeling all zeros. (Set up "window" by possible rational roots. Use ZOOM "1: ZBox " to get a better view of the zeros.)

II. Finding the Zeros of a Polynomial Function

EX. $f(x) = 2x^3 + 5x^2 - 28x - 15$

STEP 1: # of zeros: ____________ **Maximum # of turning points:** ____________

STEP 2: End behavior: ____________

STEP 3:

Possible Rational Roots (PRR): = $\dfrac{\textbf{Divisors of the constant}}{\textbf{Divisors of the leading coefficient}}$ = _________________

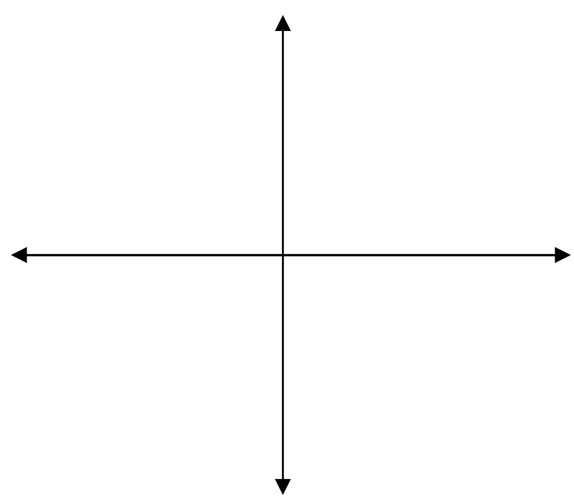
List all of the PPR: ____

STEP 4: Use your calculator table to test PRR. What are the real rational zeros?___________

STEP 5: Use synthetic division to put *f(x)* in completely factored form and find any other zeros.

$f(x) = 2x^3 + 5x^2 - 28x - 15 =$ ______________________________

STEP 6: Sketch.



EX.

$$f(x) = 3x^5 - 12x^4 - 30x^3 + 84x^2 - 45x = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Factor out common factor before using Rational Root Test!

STEP 1: # of zeros: _____________ **Maximum # of turning points:** _____________

STEP 2: End behavior: _____________

STEP 3: ***Factor out common factor before using Rational Root Test!***

Possible Rational Roots (PRR): = $\dfrac{\textbf{Divisors of the constant}}{\textbf{Divisors of the leading coefficient}}$ = ___________________

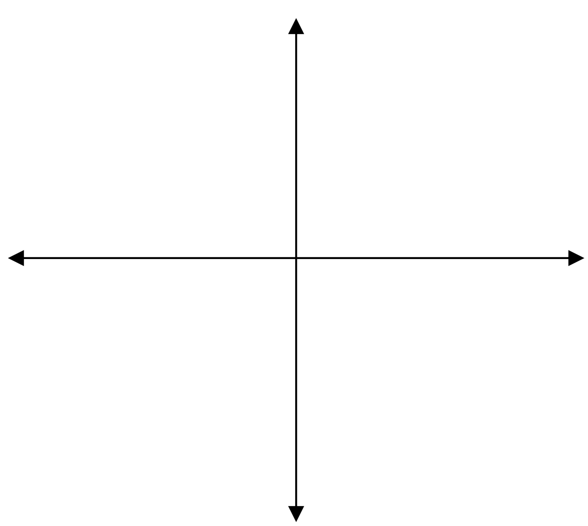
List all of the PPR: ____

STEP 4: Use your calculator table to test PRR. What are the real rational zeros?____________

STEP 5: Use synthetic division to put *f(x)* in completely factored form and find any other zeros.

$$f(x) = 3x^5 - 12x^4 - 30x^3 + 84x^2 - 45x = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

STEP 6: Sketch.



EX. $f(x) = x^3 - 4x^2 + 25x - 100$

STEP 1: # of zeros: ____________ **Maximum # of turning points:** ____________

STEP 2: End behavior: ____________

STEP 3:

Possible Rational Roots (PRR): = $\dfrac{\textbf{Divisors of the constant}}{\textbf{Divisors of the leading coefficient}}$ = __________________

List all of the PPR: ____

STEP 4: Use your calculator table to test PRR. What are the real rational zeros?___________

STEP 5: Use synthetic division or factor by grouping to put *f(x)* in completely factored form and find any other zeros.

$f(x) = x^3 - 4x^2 + 25x - 100$ = ______________________________

STEP 6: Sketch.