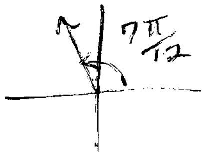
Math 112 Name: Answer Key

Practice Final Exam

See the Chapter Reviews for summaries of basic information and additional problems!!
Show all your work. Partial credit is based on work shown!

1. a. Draw a sketch of the angle $\frac{7\pi}{12}$.

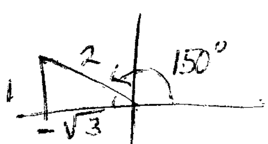


b. What is this angle expressed in degrees?

$\frac{7\pi}{12}\left(\frac{180}{\pi}\right) = 105^{\circ}$

2. **Evaluate** (without a calculator) giving the **exact value** for each of the following. (Draw and label the sides of the reference triangle.)

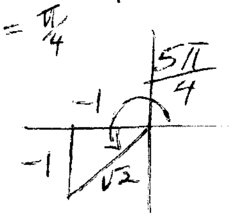
a. $\cos 150^{\circ} = \underline{-\frac{\sqrt{3}}{2}}$



reference ∠ = 30°

b. $\sin \frac{5\pi}{4} = \underline{-\frac{\sqrt{2}}{2}}$

ref ∠ = $\frac{\pi}{4}$

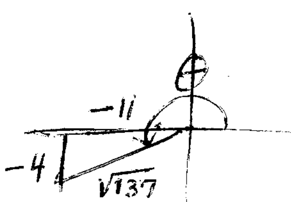


$\sin \frac{5\pi}{4} = -\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= -\frac{\sqrt{2}}{2}$

3. a. If $\tan\theta = \frac{4}{11}$ and $\cos\theta < 0$, angle θ is in what quadrant? $\underline{III}$

b. Draw and label the sides of the reference triangle.



c. Determine the remaining five trigonometric functions of θ.

$\cot\theta = \frac{11}{4}$

$\sin\theta = \frac{-4}{\sqrt{137}} \cdot \frac{\sqrt{137}}{\sqrt{137}} = -\frac{4\sqrt{137}}{137}$

$\cos\theta = -\frac{11}{\sqrt{137}}$ or $-\frac{11\sqrt{137}}{137}$

$\csc\theta = \frac{\sqrt{137}}{4}$

$\sec\theta = -\frac{\sqrt{137}}{11}$

d. $\sin 2\theta = 2\sin\theta\cos\theta$

$\sin 2\theta = 2\left(\frac{-4}{\sqrt{137}}\right)\left(\frac{-11}{\sqrt{137}}\right)$

$\sin 2\theta = +\frac{88}{137}$

e. $\sin(\theta/2)$

$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$

$\sin\frac{\theta}{2} = \sqrt{\frac{1 - \frac{-11}{\sqrt{137}}}{2}}$

$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1}{2}\left(1+\frac{11}{\sqrt{137}}\right)}$

$\sin\frac{\theta}{2} \approx .9848$

Note: θ is in Quad III

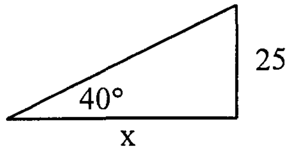
thus $\pi < \theta < \frac{3\pi}{2}$

$\frac{\pi}{2} < \frac{\theta}{2} < \frac{3\pi}{4}$

so $\frac{\theta}{2}$ is in quad II

thus $\sin\frac{\theta}{2}$ is +

4. Solve for x.



$\tan 40^{\circ} = \frac{25}{x}$

$x \tan 40^{\circ} = 25$

$x = \frac{25}{\tan 40^{\circ}} = \frac{25}{.8391} = 29.79$ units

or

$\cot 40^{\circ} = \frac{x}{25}$

$25(\cot 40^{\circ}) = x$

$29.79 = x$

5. Evaluate without a calculator. (Give answers in radian measure in terms of π.) $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

a. $\sin^{-1} 1$

$\sin\theta = 1$

$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$\boxed{\theta = \frac{\pi}{2}}$

b. $\cos^{-1}(-\frac{1}{2})$

$0 \le \theta \le \pi$

$\cos\theta = -\frac{1}{2}$

$\boxed{\theta = \frac{2\pi}{3}}$

c. $\tan^{-1}(-1)$

$\tan\theta = -1$

$\boxed{\theta = -\frac{\pi}{4}}$

6. Find the exact value of this expression. (Draw and label the sides of a right triangle.)

$\boxed{\tan(\cos^{-1}\left(\frac{-4}{5}\right)) = -\frac{3}{4}}$

$\cos\theta = -\frac{4}{5}$

$0 \le \theta \le \pi$

$b^2 + (-4)^2 = 5^2$

$b^2 = 25 - 16 = 9$

$b = 3$

7. For each of the following functions, graph at least two periods (one period in the positive x direction and one period in the negative x direction.) Find the pertinent information (amplitude, period, divisions of period, etc.) **Label the axes with appropriate values.** Asymptotes should be dashed lines.

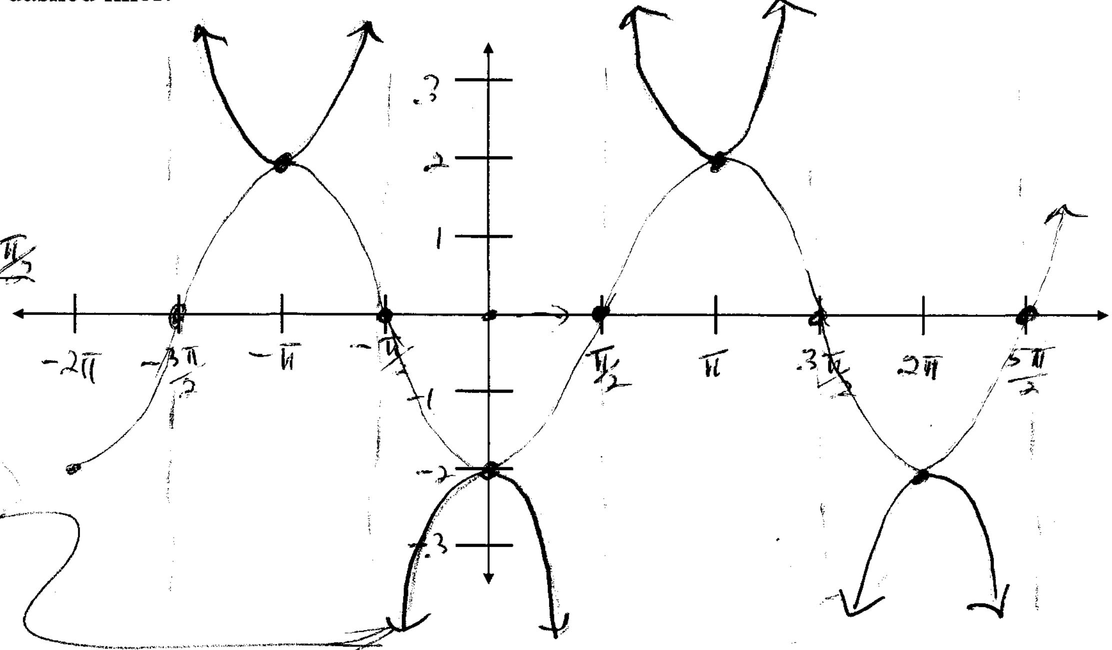
a. $y = 2\sin(x - \frac{\pi}{2})$

period: 2π

amplitude: 2

phase shift : right $\frac{\pi}{2}$

b. $y = 2\csc(x - \frac{\pi}{2})$



8. Use the sum or difference identities to write the expression as a function of a single angle. Then give the exact value of the trigonometric function.

a. $\frac{\tan 100° - \tan 40°}{1 + \tan 100° \tan 40°} = \tan 60°$

$\sqrt{3}$

b. $\sin 35° \cos 25° + \cos 35° \sin 25° = \sin 60° = \frac{\sqrt{3}}{2}$

9. Use the sum or difference formulas or half angle formulas to determine the **exact value** of the $\sin 15°$.

$\sin(60° - 45°) = \sin 60° \cos 45° - \cos 60° \sin 45°$

$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \frac{1}{4}(\sqrt{6} - \sqrt{2})$

$\cot\theta = \frac{1}{\tan\theta}$

10. Verify the following identities.

a. $(2 \sin x \cos x) \sec 2x = \tan 2x$

$\sin 2x \sec 2x$ | "

$\sin 2x \frac{1}{\cos 2x}$ | "

$\frac{\sin 2x}{\cos 2x}$ | "

$\tan 2x$ | "

b. $\tan\theta\cot\theta - \sin^2\theta = \cos^2\theta$

$1 - \sin^2\theta =$ "

$\cos^2\theta =$ "

$0 \le 2\theta < 4\pi$

11. Solve the following trigonometric equations in the interval $0 \le \theta < 2\pi$.

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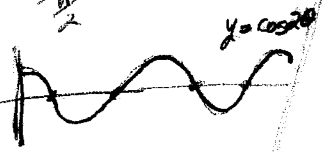
a. $\cos(2\theta)=0$

$\frac{\pi}{2}$

$\frac{3\pi}{2}$

$2\theta = \frac{\pi}{2} + \frac{3\pi}{2} + 2k\pi$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$



b. $2\sin^2 x + 3\cos x - 3 = 0$

$2(1-\cos^2 x) + 3\cos x - 3 = 0$

$2 - 2\cos^2 x + 3\cos x - 3 = 0$

$0 = 2\cos^2 x - 3\cos x + 1$

$0 = (2\cos x - 1)(\cos x - 1)$

$2\cos x - 1 = 0$ or $\cos x - 1 = 0$

$\cos x = \frac{1}{2}$ $\quad$ $\cos x = 1$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$ $\quad$ $x = 0$

12. Solve the triangle for all sides and angles not given. If two solutions exist, find both.
A = 58° , a = 4.5 inches, and b= 5 inches.

$\frac{\sin 58°}{4.5} = \frac{\sin B}{5}$

$\sin B = \frac{5 \sin 58°}{4.5}$

$\sin B \approx .9423$

$B = \sin^{-1}(.9423)$

$B_1 \approx 70.4°$ or $109.6° \approx B_2$

$\gamma_1 = 180 - (\alpha + B)$

$\gamma_1 = 180 - (128.4)$

$\gamma_1 = 51.6°$

or

$\gamma_2 = 180 - (58 + 109.6)$

$\gamma_2 = 180 - (167.6)$

$\gamma_2 = 12.4°$

$\frac{\sin 58°}{4.5} = \frac{\sin 51.6}{c_1}$

$c_1 = \frac{(\sin 51.6) 4.5}{\sin 58°}$

$c_1 \approx 4.2$

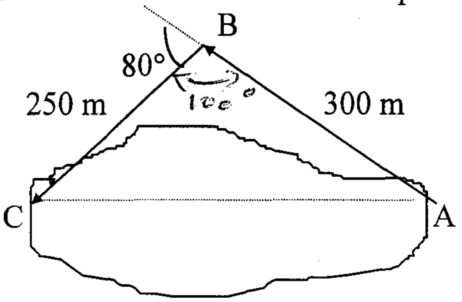
or

$\frac{\sin 58°}{4.5} = \frac{\sin 12.4}{c_2}$

$c_2 = \frac{(\sin 12.4) 4.5}{\sin 58}$

$c_2 = 1.14$

13. To approximate the length of a marsh, a surveyor walks 300 meters from point A to point B, then turns 80° and walks 250 meters to point C. Calculate the approximate the length AC across the marsh.



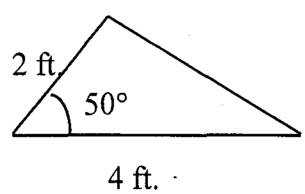
$b^2 = a^2 + c^2 - 2ac\cos B$

$b^2 = (250)^2 + (300)^2 - [2(250)(300)(\cos 100°)]$

$b^2 = 178547.2267$

$b \approx 422.5$ meters

14. Find the area of this triangle.



$A = \frac{1}{2} a b (\sin \gamma)$

$A = \frac{1}{2}(2)(4)(\sin 50°)$

$A = 4 \sin 50°$

$A \approx 3.064$ sq ft.

16.a. Continue this sequence by filling in the next three terms.

+5 +5 +5

6, 11, 16, 21, 26, 31, 36, 41, 46

b. Write the nth term of this sequence. That is, write a formula for a_n.

$a_n = 6 + (n-1)5$ or $5n+1$

c. What is the 50th term in this sequence?

$a_{50} = 6 + (49)5 = 251$

or $a_{50} = 5(50) + 1 = 251$

d. Find the sum of the first fifty terms.

$S_{50} = \frac{50}{2}(6 + 251)$

$= 25(257) = 6425$

17. Write the first five terms of the sequence defined by $a_1 = 25$ and $a_{k+1} = -\frac{3}{5}a_k$.

$a_1 = 25$

$a_2 = (-\frac{3}{5})(25) = -15$

$a_3 = (-\frac{3}{5})(-15) = +9$

$a_4 = (-\frac{3}{5})(9) = -5.4$

$a_5 = (-\frac{3}{5})(-5.4) = 3.24$

18. Find the sum. $\sum_{k=1}^{\infty} 4\left(\frac{2}{3}\right)^{k-1}$

$a_1 = 4(\frac{2}{3})^0 = 4$

$r = \frac{2}{3}$

$a_2 = 4(\frac{2}{3})^1 = \frac{8}{3}$

etc

$S_\infty = \frac{a}{1-r}$

$S_\infty = \frac{4}{1-\frac{2}{3}} = \frac{4}{+\frac{1}{3}} = 12$