Fri, March 11th

Math 112, section ______ ______ NAME: Answers

Test 2, version A, Spring 2011 100

Sections 7.6-7.8 & 8.1-8.5 Seat Location: ________

Show all your work. Full credit is based on your work shown!

6 pts (2pts each blank)

1. a. Use a sum or difference identity to write the expression as a function of a single angle, then find the **exact value** of the expression. cos 165° cos 15° − sin 165° sin 15° = $\cos(165+15)$

$= \cos 180° = -1$

10 pts

b. Use a sum or difference identity to find the **exact value** of $\cos\frac{7\pi}{12}$.

2pts

$$\cos\left(\frac{3\pi}{12}+\frac{4\pi}{12}\right)=\cos\left(\frac{\pi}{4}+\frac{\pi}{3}\right)=\boxed{\cos\frac{\pi}{4}\cos\frac{\pi}{3}-\sin\frac{\pi}{4}\sin\frac{\pi}{3}}$$ 2pts

$$=\boxed{\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)-\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)}$$ 4pts

$$=\frac{\sqrt{2}}{4}-\frac{\sqrt{6}}{4}=\boxed{\frac{1}{4}\left(\sqrt{2}-\sqrt{6}\right)}$$ 2pts

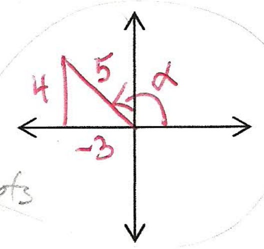
or $\boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$

2. Given that $\sin\alpha=\frac{4}{5}$ *with* $\frac{\pi}{2}<\alpha<\pi$, *and* $\sin\beta=-\frac{2}{\sqrt{5}}=-\frac{2\sqrt{5}}{5}$, *with* $\pi<\beta<\frac{3\pi}{2}$,

find the **exact value** of each of the following: *(Sketch a reference triangle and label its sides.)*

5 pts

a. $\cos\alpha = -\frac{3}{5}$ 2pts

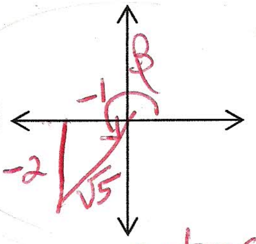


3pts

5 pts

b. $\cos\beta = -\frac{1}{\sqrt{5}}$ 2pts

or $= -\frac{\sqrt{5}}{5}$



3pts

notes:

$\pi<\beta<\frac{3\pi}{2}$

$\frac{\pi}{2}<\frac{\beta}{2}<\frac{3\pi}{4}$

quad II

6 pts

c. $\sin(\alpha+\beta) = \sin\alpha\cos\beta+\cos\alpha\sin\beta$

4pts $= \frac{4}{5}\left(-\frac{\sqrt{5}}{5}\right)+\left(-\frac{3}{5}\right)\left(-\frac{2\sqrt{5}}{5}\right)$

2pts $= -\frac{4\sqrt{5}}{25}+\frac{6\sqrt{5}}{25} = \boxed{+\frac{2\sqrt{5}}{25}}$

6 pts

d. $\sin(2\alpha) = 2\sin\alpha\cos\alpha$

$= 2\left(\frac{4}{5}\right)\left(-\frac{3}{5}\right)$ 3pts

$= -\frac{24}{25}$ 3pts

Note.
$\frac{\pi}{2}<\alpha<\pi$
$\pi<2\alpha<2\pi$
thus $\sin(2\alpha)$ is neg.
−1pt if not neg.

6 pts

e. $\cos\left(\frac{\beta}{2}\right) = -\sqrt{\frac{1+\cos\beta}{2}}$ 1pt

$= -\sqrt{\left(\frac{1+\left(-\frac{\sqrt{5}}{5}\right)}{2}\right)\cdot\frac{5}{5}}$ 4pts

$= \boxed{-\sqrt{\frac{5-\sqrt{5}}{10}}}$ 1pt

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$(\sin\theta + \cos\theta)^2 \neq \sin^2\theta + \cos^2\theta$

there is middle term $+ 2\sin\theta\cos\theta$

But they just happen to cancel here.

3. Establish each identity. Show all your steps to indicate which identities you used.

−1 pt if left out middle terms.

5 pts

a. $\cos\theta(\tan^2\theta + 1) = \sec\theta$

2 pts $\cos\theta(\sec^2\theta) = \sec\theta$

1 pt $\cancel{\cos\theta}\left(\frac{1}{\cos^{\cancel{2}}\theta}\right) = $ "

1 pt $\frac{1}{\cos\theta} = $ "

1 pt $\sec\theta = \sec\theta$

7 pts

b. $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$

$(\sin^2\theta + \cancel{2\sin\theta\cos\theta} + \cos^2\theta) + (\sin^2\theta - \cancel{2\sin\theta\cos\theta} + \cos^2\theta) = 2$

3

5 pts $(\sin^2\theta + \cos^2\theta) + (\sin^2\theta + \cos^2\theta) = 2$

$1 + 1 = 2$

2 pts $2 = 2$

8 pts

c. $\csc x - \sin x = \cos x \cot x$

simplify both sides till = or change till left side = rt. side.

Substi 2 pts $\frac{1}{\sin x} - \sin x = \cos x \cdot \boxed{\frac{\cos x}{\sin x}}$ 2 pts

2 pts LCD $\frac{1}{\sin x} - \frac{\sin^2 x}{\sin x} = \frac{\cos^2 x}{\sin x}$

1 pt $\frac{1 - \sin^2 x}{\sin x} = \frac{\cos^2 x}{\sin x}$

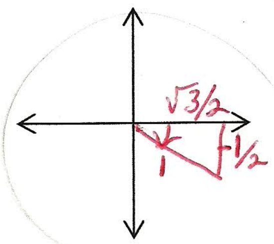
1 pt $\frac{\cos^2 x}{\sin x} = \frac{\cos^2 x}{\sin x}$

12 pts

4. Evaluate without a calculator giving **exact values**. Draw and label a sketch to illustrate each one. [Note: Your sketch should show the angle and a labeled triangle or a point on the unit circle.]

a. $\sin^{-1}\left(-\frac{1}{2}\right) = \underline{-\pi/6}$ −1 pt if $\frac{11\pi}{6}$ 2 pts

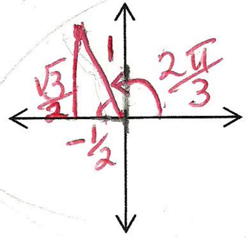
reference ∠ = $\underline{\pi/6}$ (in radians) 1 pt



2 pts

b. $\tan\left[\cos^{-1}\left(-\frac{1}{2}\right)\right] = \left(\tan\left(\frac{2\pi}{3}\right)\right) = -\sqrt{3}$ 2 pt 2 pts

reference ∠ = $\underline{\pi/3}$ (in radians) 1 pt



2 pts

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Section 2.8 $\boxed{y=A\cos(\omega x-\phi)+B}$

8pts (2 for each part)

5. Write the equation of the cosine function that satisfies the following information.

Amplitude = 1.5, period = 4π, phase shift = $\frac{\pi}{4}$ units to the right, and vertical shift = up 2 unit.

$y = 1.5\cos\left(\frac{1}{2}x-\frac{\pi}{8}\right)+2$ or $y=1.5\cos\left[\frac{1}{2}\left(x-\frac{\pi}{4}\right)\right]+2$

16pts

6. For each of the following functions, graph at least two periods (one period in the positive x direction and one period in the negative x direction.) Find the pertinent information (amplitude, period, divisions of period, etc.) **Label the axes with appropriate values**. Asymptotes should be dashed lines. Plot at least 5 points in each period.

a. $y = -3\sin(2x-\pi) = -3\sin\left(2\left(x-\frac{\pi}{2}\right)\right)$

1pt period: π

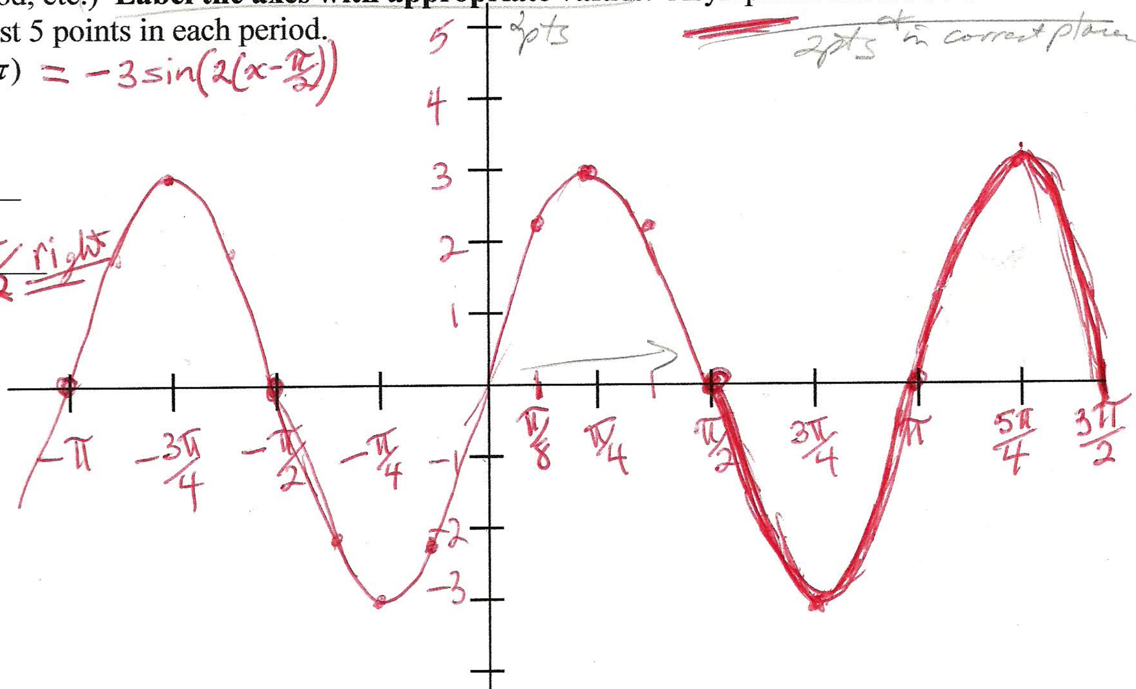
1pt amplitude: 3

2pts phase shift : $\frac{\pi}{2}$ right

2pts overall shape of graph

2pts

x	y
0	0
$\frac{3\pi}{8}$, $\frac{\pi}{8}$	2.12
$\frac{\pi}{4}$	3
$\frac{\pi}{2}$	0
$+\frac{3\pi}{4}$	-3
π	0



2pts

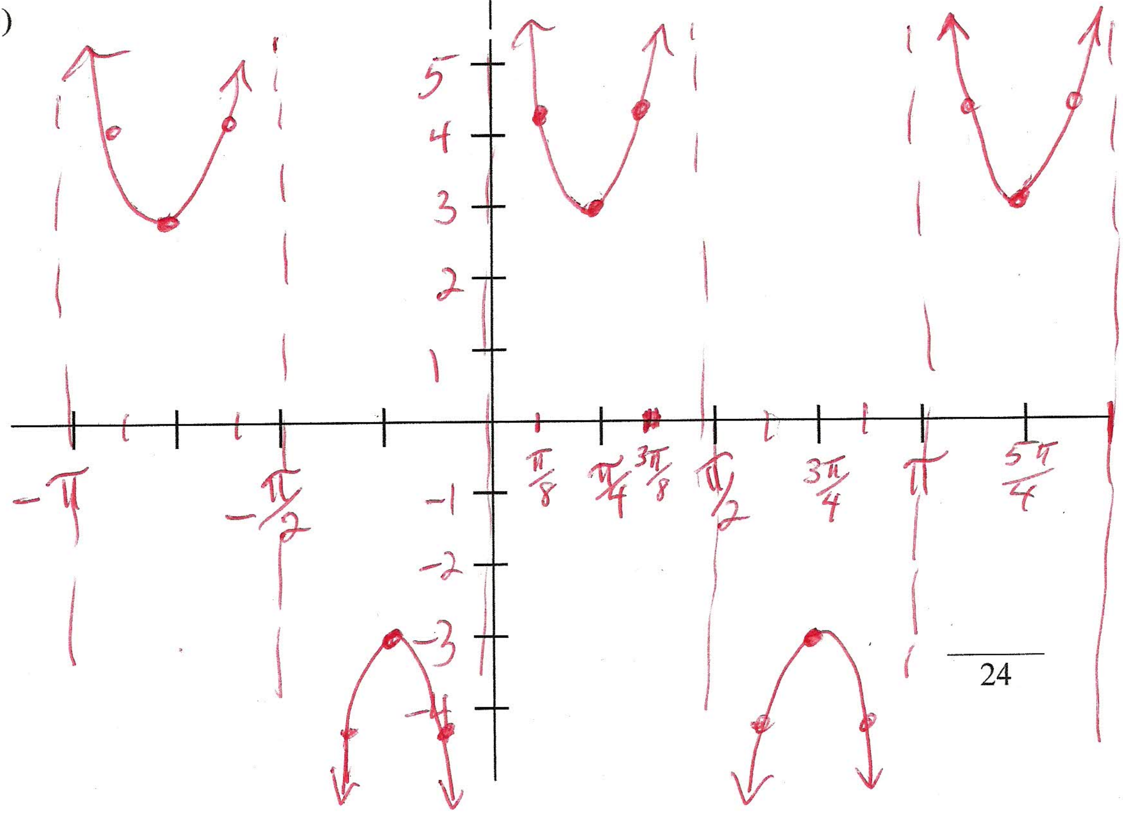
2pts at correct place

b. $y = -3\,\overset{csc}{\cancel{\sec}}(2x-\pi)$

2pts

x	y
$\pm\frac{\pi}{2}$, 0	undefined
$\frac{3\pi}{8}$, $\frac{\pi}{8}$	4.24
$\frac{\pi}{4}$	+3

2pts overall shape of graph.



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