Math 112, section ______ ______ NAME: Answers

Test 2, version A, spring 2012 100

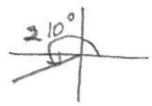
Sections 7.6-7.8 & 8.1-8.5 Seat Location: ________

Show all your work. Full credit is based on your work shown!

6 pts (2 pts per blank)

8.4

1. a. Use a sum or difference identity to write the expression as a function of a single angle, then find the **exact value** of the expression. sin 265° cos 55° – cos 265° sin 55° = $\sin(265-55)°$



$= \boxed{\sin 210°} = \boxed{-\tfrac{1}{2}}$

10 pts

b. Use a sum or difference identity to find the **exact value** of $\cos\frac{13\pi}{12}$.

$\frac{10\pi}{12}+\frac{3\pi}{12}$

$\left(\frac{5\pi}{6}+\frac{\pi}{4}\right)$

$\cos\left(\frac{13\pi}{12}\right) = \cos\left(\frac{9\pi}{12}+\frac{4\pi}{12}\right) = \cos\left(\frac{3\pi}{4}+\frac{\pi}{3}\right)$ 2pts

(there are other combinations) $= \left(\cos\frac{3\pi}{4}\right)\left(\cos\frac{\pi}{3}\right) - \left(\sin\frac{3\pi}{4}\right)\left(\sin\frac{\pi}{3}\right)$ 2pts

$= -\frac{\sqrt{2}}{2}\cdot\frac{1}{2} - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$ 4pts

OK if $\left(\frac{-\sqrt{2}-\sqrt{6}}{4}\right)$

$= -\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = -\frac{1}{4}(\sqrt{2}+\sqrt{6})$ 2pts

24pts

see 8.5 Ex6

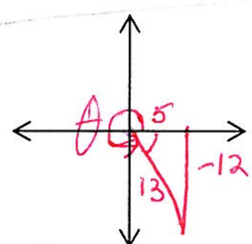
2. If $\sin\theta = -\frac{12}{13}$, *with* $\frac{3\pi}{2} < \theta < 2\pi$, then θ is in quadrant 4 or IV and $\left(\frac{\theta}{2}\right)$ is in quadrant 2 or II. 1pt 1pt

$\frac{3\pi}{4} < \frac{\theta}{2} < \pi$

Find the **exact value** of each of the following:
(sketch a reference triangle and label its sides.)

a. $\cos\theta = \frac{5}{13}$

2pts



3pts

b. $\cos\left(\frac{\theta}{2}\right) = -\sqrt{\frac{1+\cos\theta}{2}}$ 1pt

$= -\sqrt{\frac{1+\frac{5}{13}}{2}\cdot\frac{13}{13}}$

4pts

$= -\sqrt{\frac{13+5}{26}} = -\sqrt{\frac{18}{26}} = -\sqrt{\frac{9}{13}}$

1pt $= \boxed{-\frac{3}{\sqrt{13}}}$ or $\boxed{\frac{-3\sqrt{13}}{13}}$

c. $\sin(2\theta) = 2\sin\theta\cos\theta$

$= 2\left(-\frac{12}{13}\right)\left(\frac{5}{13}\right)$

3pts

$= -\frac{120}{169}$

2pts

6pts d. $\cos(2\theta) = 1-2\sin^2\theta$

$= 1-2\left(-\frac{12}{13}\right)^2 = 1-2\left(\frac{144}{169}\right)$

$= 1-\frac{288}{169} = -\frac{119}{169}$

or $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\frac{5}{13}\right)^2 - \left(\frac{-12}{13}\right)^2$

$= \frac{25}{169} - \frac{144}{169} = -\frac{119}{169}$

40

3. Establish each identity. Show all your steps to indicate which identities you used.

4 pts

a. $\sin\alpha\csc\alpha - \cos^2\alpha \overset{?}{=} \sin^2\alpha$

8,3 #26

$\sin\alpha \cdot \frac{1}{\sin\alpha} - \cos^2\alpha \overset{?}{=} \sin^2\alpha$

$\boxed{1 - \cos^2\alpha} \overset{?}{=} \sin^2\alpha$

$\sin^2\alpha \overset{\checkmark}{=} \sin^2\alpha$

8 pts

8,3 #

b. $\dfrac{\tan\theta + \cot\theta}{\sec\theta\csc\theta} = 1$

or mult by $\cos\theta\sin\theta$

4pts

$\dfrac{\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)}{\frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta}} \cdot \left(\dfrac{\cos\theta \cdot \sin\theta}{\cos\theta \cdot \sin\theta}\right) \Rightarrow \dfrac{\sin^2\theta + \cos^2\theta}{1}$

$= \dfrac{1}{1}$

$\dfrac{\dfrac{\sin^2\theta + \cos^2\theta}{(\cos\theta)(\sin\theta)}}{\dfrac{\sin\theta \cdot \cos\theta}{\cos\theta\sin\theta}} =$

4pts

$\dfrac{1}{1} =$

$1 \overset{\checkmark}{=} 1$

(8.3 #29)

8 pts Mult.

c. $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) \overset{?}{=} 1$

$\sec^2\theta - \cancel{\tan\theta\sec\theta} + \cancel{\tan\theta\sec\theta} - \tan^2\theta$

or

4pts $(\sec^2\theta) - \tan^2\theta \overset{?}{=} 1$

2pts $(\tan^2\theta + 1) - \tan^2\theta = 1$

$1 \overset{\checkmark}{=} 1$

2pts

or (Convert to sin and cos)

$\left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)\left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)$

$\left(\frac{1+\sin\theta}{\cos\theta}\right)\left(\frac{1-\sin\theta}{\cos\theta}\right)$

$\frac{(1-\sin^2\theta)}{\cos^2\theta} = \frac{\cos^2\theta}{\cos^2\theta} = 1$

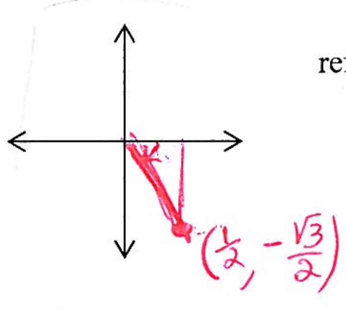
12 pts

8,1-2

4. Evaluate without a calculator giving **exact values**. Draw and label a sketch to illustrate each one. [Note: Your sketch should show the angle and a labeled triangle or a point on the unit circle.]

a. $\tan^{-1}\left(-\sqrt{3}\right) =$ $-\frac{\pi}{3}$ 2pts

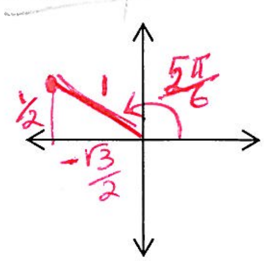
reference $\angle =$ $\frac{\pi}{3}$ 1pt
(in radians)



2pts

b. $\sin\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right] = \sin[\frac{5\pi}{6}] = \frac{1}{2}$ 2pts 2pts

reference $\angle =$ $\frac{\pi}{6}$ 1pt
(in radians)



2pts

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7.6–7.8

-2 pt if have $(2x+\frac{\pi}{2})$

8pts (2 pts each part)

5. Write the equation of the cosine function that satisfies the following information.

Amplitude = 3, period = π, phase shift = $\frac{\pi}{2}$ units to the left, and vertical shift = up 4 unit.

$y = 3\cos 2\left(x + \frac{\pi}{2}\right) + 4$ or $y = 3\cos\left(2x + \pi\right) + 4$

20pts

6. For each of the following functions, graph at least two periods (one period in the positive x direction and one period in the negative x direction.) Find the pertinent information (amplitude, period, phase shift, x-scale, etc.) **Label the axes with appropriate values.** Asymptotes should be dashed lines. Plot the "critical" points in each period.

x-scale = $\pi/8$ — 2pts for axes labeled

4pts for graph that accurately shows this information

a. $y = -2\sin(4x - \pi)$ or $y = -2\sin 4\left(x - \frac{\pi}{4}\right)$

1 pt period: $\frac{\pi}{2}$

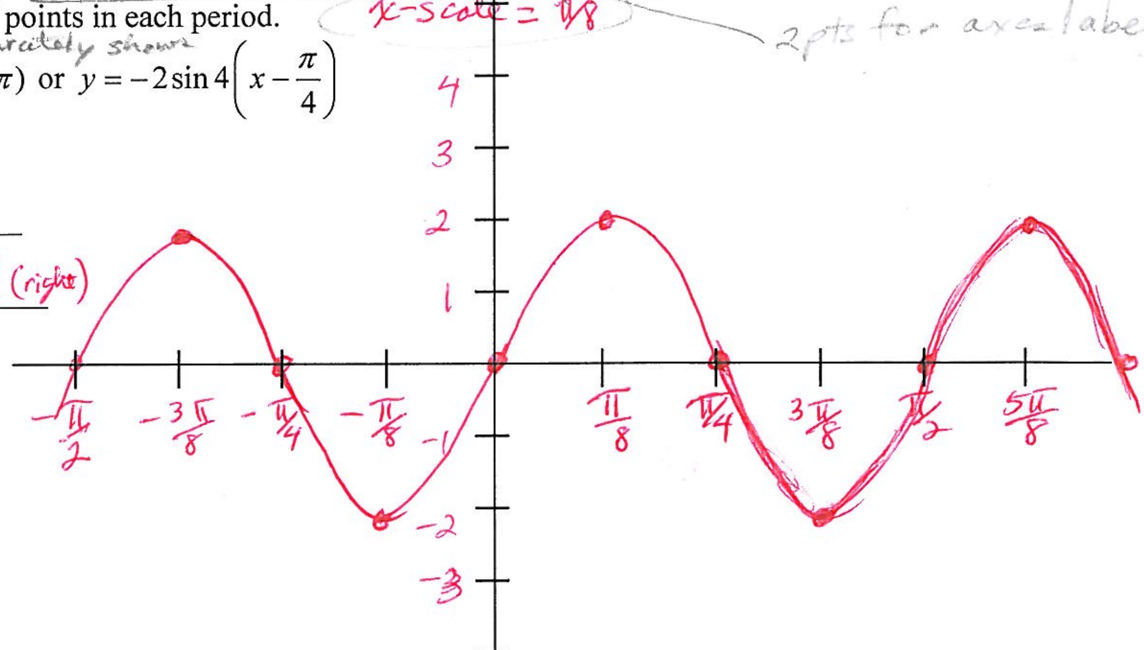
1 pt amplitude: 2

1 pt phase shift : $\frac{\pi}{4}$ (right)

3pts for any approp. pts.

x	y
$\pm\frac{\pi}{2}, \pm\frac{\pi}{4}$	0
$-\frac{3\pi}{8}, \frac{\pi}{8}$	2
$-\frac{\pi}{8}, \frac{3\pi}{8}$	-2

$(\pi/16 \approx 1.4)$



b. $y = -2\csc(4x - \pi)$ or $y = -2\csc 4\left(x - \frac{\pi}{4}\right)$

(Don't take off pts if this is wrong, but is OK based on their wrong graph for part a)

2pts for any appropriate pts

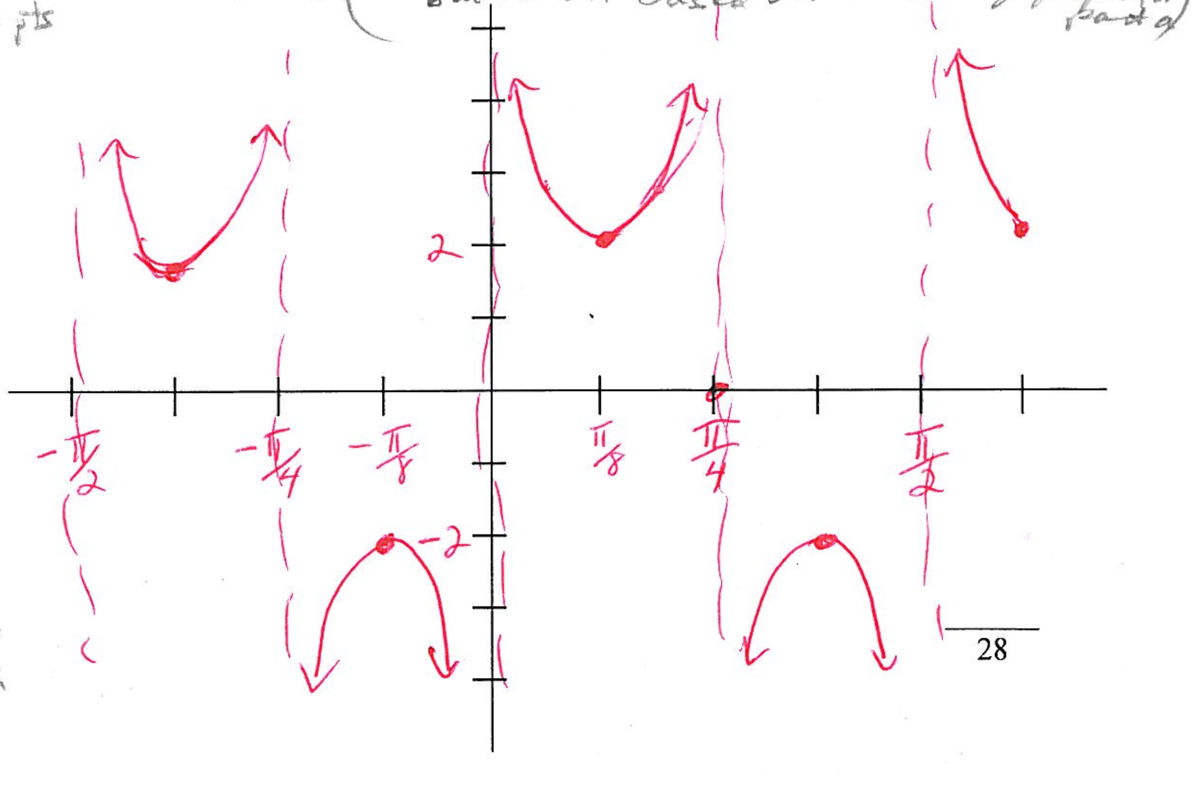
x	y
$\pm\frac{\pi}{2}, \pm\frac{\pi}{4}$	asymptotes (undefined)
$-\frac{3\pi}{8}, \frac{\pi}{2}$	2
$-\frac{\pi}{8}, \frac{3\pi}{8}$	-2

$(\frac{\pi}{16} \approx 2.8)$

2pts for asymptotes drawn

2pts for axes labeled

2pts for overall shape + accuracy of graph



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