MAT 112 -
13.1 – 13.3

Group Work

Name: Answers
Seat #: ______ - ______

Write down the first 5 terms of each sequence. Thus $n = 1, 2, 3, 4, 5$

1. $\{(n-1)^2\}$ $= 0^2, 1^2, 2^2, 3^2, 4^2 = 0, 1, 4, 9, 16$

2. $\{(-1)^n(\frac{n+3}{n+1})\}$ $= -\frac{4}{2}, +\frac{5}{3}, -\frac{6}{4}, +\frac{7}{5}, -\frac{8}{6}$

$$\sum_{k=1}^{n} c = \overbrace{c + c + c + \cdots + c}^{n \text{ terms}} = cn$$

$$\sum_{k=1}^{n} k = 1+2+3+\ldots+n = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = 1^2+2^2+3^2+\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = 1^3+2^3+3^3+\ldots+n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Evaluate using formulas. or use sum(seq(formula, variable, beg.value, end.value, step)

3. $\sum_{k=1}^{10}(k+3) =$ 85

$$\sum_{k=1}^{10} k + \sum_{k=1}^{10} 3$$

$$\frac{10(11)}{2} + 3(10)$$

$$5(11) + 30 = 55 + 30 = 85$$

4. $\sum_{k=1}^{8}(2k^2-1) =$ 400

$$2\sum_{k=1}^{8} k^2 - \sum_{k=1}^{8} 1 = 2\left[\frac{8(9)(17)}{6}\right] - 1(8)$$

$$= 408 - 8 = 400$$

$\boxed{\text{Sum(seq}(2x^2-1, x, 1, 8, 1) = 400}$

Find a general formula for the sequence.

5. $\{3, -5, 7, -9, 11, -13, \ldots\}$ $a_n =$ $(-1)^{(n+1)}(2n+1)$

Notice the pattern of odd numbers, beginning with 3.
Also notice the alternating signs, beginning with positive.

Arithmetic Sequences	**Geometric Sequences**
$a_n = a_1 + d(n-1)$	$a_n = a_1 r^{n-1}$
$S_n = \frac{n}{2}(a_1 + a_n)$	$S_n = \frac{a_1(1-r^n)}{1-r}$
	$S_\infty = \frac{a_1}{1-r}, \lvert r\rvert < 1$

6. a. Write out the terms in the indicated sequence and then find the sum.

Add to find sum

if k=1 if k=2 if k=3 if k=4 if k=5 k=6 k=7

$\sum_{k=1}^{7} 4k-3 = 1 + 5 + 9 + 13 + 17 + 21 + 25$ (4 pts.) $= \boxed{91}$

Use formula $S_7 = \frac{7}{2}(1+25) = \frac{7}{2}(26) = 91$

b. Is this an arithmetic sequence or a geometric sequence? arithmetic (2 pts.)

c. Find the 35th term of the sequence. $a_{35} =$ 137 (4 pts.)

$a_{35} = a_1 + (n-1)d$

$a_{35} = 1 + (34)(4) = 1 + 136 = 137$

or

$a_{35} = 4n - 3 = 4(35) - 3 = 140 - 3 = 137$

d. Find the sum of the first 35 terms of the sequence. $S_{35} =$ 2415 (4 pts.)

$S_{35} = \frac{35}{2}(1 + 137) = \frac{35}{2}(138)$

7. Given the sequence $\frac{10}{1}, \frac{10}{3}, \frac{10}{9}, \frac{10}{27}, \ldots$

Notice: $a_1 = 10$, $r = \frac{1}{3}$ & thus $|r| < 1$

a. Is the sequence arithmetic or geometric? Geometric (2 pts.)

b. Write a formula for the nth term of the sequence. $a_n =$ $10/3^{(n-1)}$ (4 pts.)

$a_n = \left(\frac{10}{1}\right)\left(\frac{1}{3}\right)^{n-1}$ or $\frac{10}{3^{(n-1)}}$

c. What is the 8th term? $a_8 =$ $10/3^7 = \frac{10}{2187}$ (4 pts.)

$a_8 = \left(\frac{10}{1}\right)\left(\frac{1}{3}\right)^7$ or $\frac{10}{3^7}$

d. What is the sum of the first 8 terms? $S_8 =$ 14.99771376 (4 pts.) $\frac{15(6560)}{6561}$

$S_8 = \frac{10(1-(\frac{1}{3})^8)}{1-\frac{1}{3}} = 10\left(1-\frac{1}{3^8}\right)\left(\frac{3}{2}\right)$

e. What is the sum of this infinite sequence? $S_n =$ 15 (4 pts.)

$S_\infty = \frac{10}{1-\frac{1}{3}} = \frac{10}{\frac{2}{3}} = 10 \cdot \frac{3}{2} = \frac{30}{2} = 15$

or Sum(seq(10/3^(n-1), n, 1, 8)) = 14.99771376

(or variable of X if in function mode)

note: as $n \to \infty$, $S_\infty \to 15$