10.3 The Complex Plane; DeMoivre's Theorem

In Review:

$i =$ ______

$i^2 =$ ______

$i^3 =$ ______

$i^4 =$ ______

$i^5 =$ ______

$i^6 =$ ______

Complex # in standard form (rectangular form):

$z = a + bi$ or $z = x + yi$

EX: Solve the following quadratic equation.

$$2x^2 + x + 4 = 0$$

I. Operations on Complex Numbers in Standard Form

$z_1 = 2 + 3i$

$z_2 = 4 - 5i$

Addition $z_1 + z_2 =$

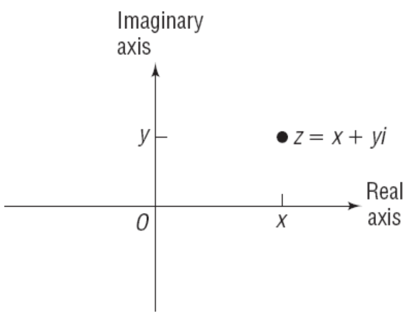
Subtraction $z_1 - z_2 =$

Multiplication $z_1 \bullet z_2 =$

Division $\frac{z_1}{z_2} =$

II. Geometric Representation of a Complex Number

Complex plane



Graph the following:

$z = 2 + 4i$

$z = 1 - 3i$

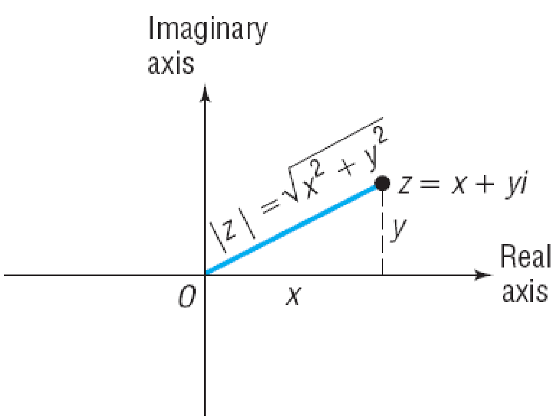
$z = -4 + 2i$

$z = 3 + 0i$

$z = 0 - 2i$

Let $z = x + yi$ be a complex number. The **magnitude** or **modulus** of z, denoted by $|z|$, is defined as the distance from the origin to the point (x, y). That is,

$$|z| = \sqrt{x^2 + y^2}$$



Pythagorean Theorem: $x^2 + y^2 = r^2$

θ is called the **argument**

tan θ

sin θ

cosθ

Standard Form of a Complex Number:

Polar Form of a Complex Number:

Convert from Standard Form to Polar Form

EX: $z = 1 - 1i$

EX: $z = -2 + 3i$

Convert from Polar Form to Standard Form

EX: $z = 2[\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}]$

EX: $z = 3[\cos 22^\circ + i\sin 22^\circ]$

III. Multiplying and Dividing Complex Numbers in Polar Form

If $r \geq 0$ and $0 \leq \theta < 2\pi$, the complex number $z = x + yi$ may be written in **polar form** as

$$z = x + yi = (r\cos\theta) + (r\sin\theta)i = r(\cos\theta + i\sin\theta) \qquad \textbf{(4)}$$

Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$ be two complex numbers. Then

$$z_1 z_2 = r_1 r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)] \qquad \textbf{(5)}$$

If $z_2 \neq 0$, then

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)] \qquad \textbf{(6)}$$

EX:

$$z_1 = 3[\cos 20^\circ + i\sin 20^\circ]$$
$$z_2 = 5[\cos 100^\circ + i\sin 100^\circ]$$

a) $z_1 \bullet z_2 =$

b) $\dfrac{z_1}{z_2} =$

IV. DeMoivre's Theorem

De Moivre's Theorem

If $z = r(\cos\theta + i\sin\theta)$ is a complex number, then

$$z^n = r^n[\cos(n\theta) + i\sin(n\theta)]$$

where $n \geq 1$ is a positive integer.

EX: $z = 2[\cos 20^\circ + i\sin 20^\circ]$

$z^3 =$

EX: $z = 1 - i$

$z^{11} =$

V. Complex Roots Theorem

Finding Complex Roots

Let $w = r(\cos\theta_0 + i\sin\theta_0)$ be a complex number and let $n \geq 2$ be an integer. If $w \neq 0$, there are n distinct complex roots of w, given by the formula

$$z_k = \sqrt[n]{r}\left[\cos\left(\frac{\theta_0}{n} + \frac{2k\pi}{n}\right) + i\sin\left(\frac{\theta_0}{n} + \frac{2k\pi}{n}\right)\right] \qquad \textbf{(8)}$$

where $k = 0, 1, 2, \ldots, n-1$.

EX: Find the four 4$^{\text{th}}$ roots of $z = \sqrt{3} - i$