MAT 162
LabTest 4 Solution Key

Approximate the value of this series using the integral method:

$$\sum_{n=1}^{\infty} \frac{1}{3\,n^2+1}$$

By trial-and-error experimentation, we discover n = 129 is the smallest n resulting in an error smaller than .00001.

```
> restart:n:=129;.5*Int(1/(3*x^2+1),x=n..n+1);halflength:=evalf(%)
;
```

$$n := 129$$

$$0.5 \int_{129}^{130} \frac{1}{3\,x^2+1}\,dx$$

$$halflength := 0.993818449010^{-5}$$

The partial sum S_{129} is:

```
>
> n:='n';Sum(1/(3*n^2+1),n=1..129);S129:=evalf(%);
```

$$n := n$$

$$\sum_{n=1}^{129} \frac{1}{3\,n^2+1}$$

$$S129 := 0.4538528027$$

Next, we need the left and right endpoints of the interval containing the sum, so we can calculate the midpoint.

```
> S129+Int(1/(3*x^2+1),x=130..infinity);left:=evalf(%);
```

$$0.4538528027 + \int_{130}^{\infty} \frac{1}{3\,x^2+1}\,dx$$

$$left := 0.4564168884$$

```
> S129+Int(1/(3*x^2+1),x=129..infinity);right:=evalf(%);
```

$$0.4538528027 + \int_{129}^{\infty} \frac{1}{3\,x^2+1}\,dx$$

$$right := 0.4564367648$$

The midpoint is our desired approximation.

```
> midpoint:=(left+right)/2;
```

$$midpoint := 0.4564268266$$

As a check, this halflength should agree with our previous calculation of the error.

```
> halflength:=(right-left)/2;
```

$$halflength := 0.99382\ 10^{-5}$$

Conclusion: $\sum_{n=1}^{\infty} \frac{1}{3n^2+1}$ = .45643 +/- .00001