Cohort models

- VPA
- Gulland's method
- Pope's method
- Multispecies VPA
- Sections 7.5 and 7.6 in text

Cohort models

- **Trace the decline in abundance of groups of fish of similar age (cohorts) as they age and pass through the fishery**
- **Use the decline in cohort abundance to determine mortality rates**
- **'Re-creates' past population size and age-structure**
- **Very popular since the early 1970's, used extensively today for assessments.**

Cohort models

- **Originally used to estimate historical population size**
- **Now used to estimate F, current abundance, and recruitment**
- **First developed by Derzhavin (1922) with his work on sturgeon**

Virtual Population Analysis

- Fry (1949) applied Derzhavin's model and coined the term 'virtual population' for the historical population size, but assumed no M
- Ricker, Beverton & Holt, and Paloheimo incorporated M, and estimated F as a function of effort and q
- Murphy (1965) used a non-linear version of Baranov's catch equation to estimate F at early ages and worked forward

Virtual Population Analysis

- Gulland (1965) started with an F for oldest age group and worked backwards
- This estimate of F known as 'Terminal F'
- Moving backwards, estimates of F tend to converge toward true value
- Moving forward (Murphy), estimates tend to diverge from true value

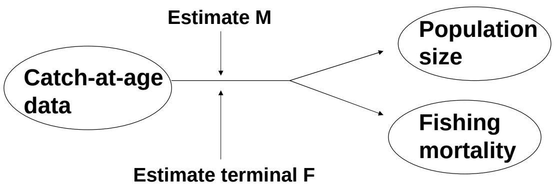
Cohort models

How they work:

- The size of a cohort when it first enters the fishery can be approximated by summing catches of that cohort across all of the years it is in fishery
- The sum of the catches is the population size that must have been present to generate the observed catches
- This 'historical population size' is known as the virtual population

Virtual Population Analysis

- **Today, we use the numbers of fish harvested to estimate <u>historic patterns</u> of F, cohort size and age-structure**
- **Referred to as cohort analysis because each cohort is analyzed separately**



Virtual Population Analysis

- **Simple relationship:**

$$\begin{pmatrix} \textit{Number alive} \\ \textit{at beginning} \\ \textit{of next year} \end{pmatrix} = \begin{pmatrix} \textit{number alive} \\ \textit{at beginning} \\ \textit{of this year} \end{pmatrix} - \begin{pmatrix} \textit{catch} \\ \textit{this} \\ \textit{year} \end{pmatrix} - \begin{pmatrix} \textit{natural} \\ \textit{mortality} \\ \textit{this year} \end{pmatrix}$$

- **If we knew initial cohort size and M, we could calculate N in each future year, but we don't…**
- **So we rearrange and work backwards**

$$\begin{pmatrix} \textit{number alive} \\ \textit{at beginning} \\ \textit{of this year} \end{pmatrix} = \begin{pmatrix} \textit{Number alive} \\ \textit{at beginning} \\ \textit{of next year} \end{pmatrix} + \begin{pmatrix} \textit{catch} \\ \textit{this} \\ \textit{year} \end{pmatrix} + \begin{pmatrix} \textit{natural} \\ \textit{mortality} \\ \textit{this year} \end{pmatrix}$$

Virtual Population Analysis

- Survivors in year t-1 = deaths due to M in year t + catch in year t + survivors in year t

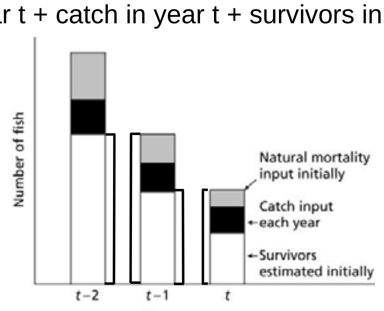


Fig. 7.9

Virtual Population Analysis methodology

Start with familiar exponential decay equation:

$$N_{t+1} = N_t e^{-(F+M)} \quad (7.7)$$

Virtual Population Analysis methodology

Start with familiar exponential decay equation:

$$N_{t+1} = N_t e^{-(F+M)} \quad (7.7)$$

and Baranov's catch equation:

$$C_t = \frac{F_t}{F_t + M} N_t \left(1 - e^{-(F_t + M)}\right) \quad (7.8)$$

Virtual Population Analysis methodology

<u>First step</u>: calculate the 'terminal' abundance by re-arranging catch equation 7.8 to give N_t

$$N_t = \frac{C_t}{(F_t / Z_t)(1 - e^{-Z_t})} \quad (7.9)$$

***** This is when we need an estimate of terminal F**

Virtual Population Analysis: methodology

2nd step: substitute a re-arranged decay equation into our catch equation to obtain F_{t-1}

$$N_{t+1} = N_t e^{-(F+M)} \quad (7.7)$$

rearranged into:

$$N_t = N_{t+1} e^{(F+M)} \quad (7.12)$$

Virtual Population Analysis: methodology

2nd step: substitute a re-arranged decay equation 7.12 into catch equation 7.8 to obtain F_t

$$N_t = N_{t+1} e^{(F+M)} \quad (7.12)$$

Substitute this for N_t below:

$$C_t = \frac{F_t}{F_t + M} N_t \left(1 - e^{-(F_t+M)}\right) \quad (7.8)$$

Virtual Population Analysis: methodology

2nd step: substitute a re-arranged decay equation 7.12 into catch equation 7.8 to obtain F_{t-1}

We end up with:

$$C_t = \frac{F_t}{F_t + M} N_{t+1} \left(e^{(F_t+M)} - 1\right) \quad (7.11)$$

Virtual Population Analysis: methodology

3rd Step: calculate N_t by inserting F_t from step 2 into re-arranged decay equation 7.12

$$N_t = N_{t+1} e^{(F_t + M)} \quad (7.12)$$

- Repeat steps 2 and 3 to work backwards through time

Virtual Population Analysis: example

For Age 3: C_3 = 80, $F_{terminal}$ = 0.6, M = 0.2

Step 1: calculate N_3

$$N_3 = \frac{C_3}{(F_3 / Z_3)(1 - e^{-Z_3})} \quad (7.9)$$

$$N_3 = \frac{80}{(0.6/0.8)(1 - e^{-0.8})} = 194$$

Virtual Population Analysis: example

For Age 2: C_2 = 90, N_3=194

Step 2: calculate F_2

$$C_2 = \frac{F_2}{F_2 + M} N_3 (e^{(F_2 + M)} - 1) \quad (7.11)$$

$$90 = \frac{F_2}{F_2 + 0.2} 194 (e^{(F_2 + 0.2)} - 1)$$

$$F_2 = 0.349$$

Virtual Population Analysis: example

For Age 2: $N_3 = 194$, $F_2=0.349$

Step 3: calculate N_2

$$N_2 = N_3 e^{(F_2+M)} \qquad (7.12)$$

$$N_2 = 194e^{(0.349+0.2)} = 335$$

- Repeat to obtain F and N for age 1

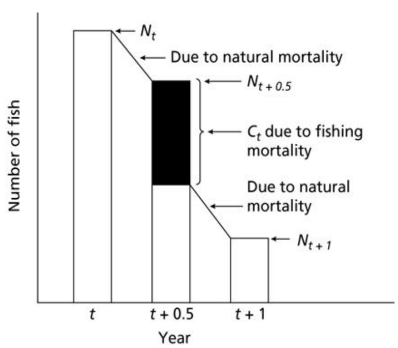
Virtual Population Analysis:

Pope's approximation for VPA

- Non-linear cohort analysis is cumbersome
- Pope (1972) proposed a step function that approximates the non-linear form
- Assumes that all fish caught exactly half-way through time period (1 year)
- Using this method, first calculate N, then F

Virtual Population Analysis:

Pope's approximation for VPA



Virtual Population Analysis:

Pope's approximation for VPA

Number of fish alive just before fishing = number alive at start of year - reduction due to half M

$$N_{t+0.5} = N_t e^{-M/2} \quad 7.13$$

Virtual Population Analysis:

Pope's approximation for VPA

Number of fish alive just before fishing = number alive a start of year -reduction due to half M

$$N_{t+0.5} = N_t e^{-M/2} \quad 7.13$$

Then all fishing occurs instantaneously

$$N_{t+0.5} = N_t e^{-M/2} - C_t \quad 7.14$$

Virtual Population Analysis:

Pope's approximation for VPA

Remaining fish suffer half M leaving number of fish alive at the end of year equal to:

$$N_{t+1} = (N_t e^{-M/2} - C_t) e^{-M/2} \quad 7.15$$

Virtual Population Analysis:

Pope's method:example

- Pope's approximation gives nearly identical result to VPA
- Method gets around iterative calculation for F
- Works best when Z < 1.0
- Can also be length-based when species cannot be aged (see 7.5.2)

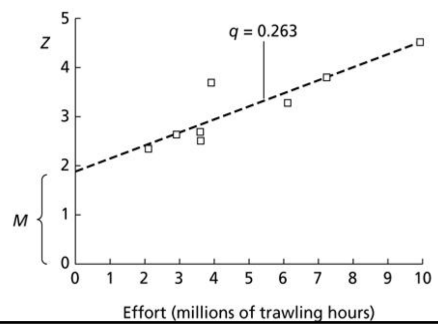
Cohort models

<u>Data requirements</u>:

- **Catch-at-age data for a long time period (8-10 years) for most fish**
- **Also need an estimate of natural mortality rate (M)**
- **Need to estimate 'terminal F' or terminal N**

Virtual Population Analysis

- How to estimate M?



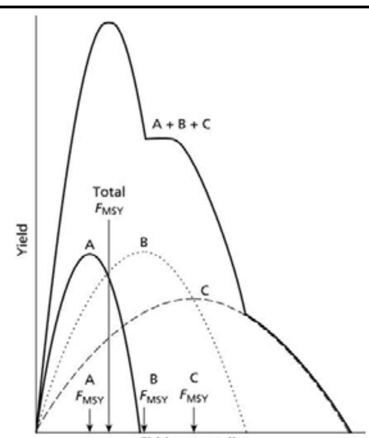
9.25

Virtual Population Analysis:
assumptions

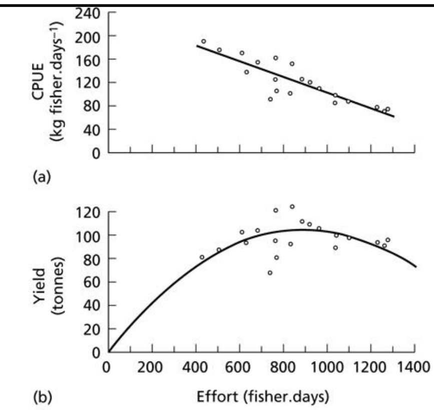
- No fish alive at some age
  - -Cohorts must have all passed thru fishery
- M is known, constant, and not very large
  - -Oldest cohorts?
  - -Best when M < F and F/Z=0.5-1
- Terminal F
  - -source of bias and model sensitivity, ‘tuning’
- No error in catch/age data
  - -Oldest cohorts? ; bycatch, discards
- No net immigration or emmigration

Virtual Population Analysis:
alternatives: Multispecies (Chap 8)

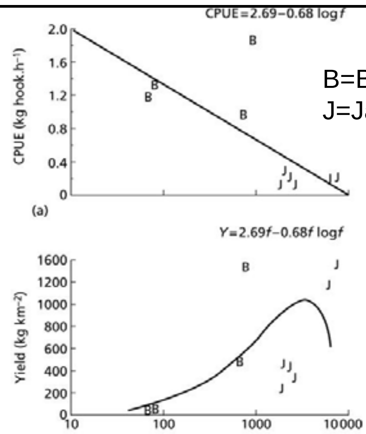
- Multispecies
  - – Surplus production (temporal, spatial)



8.1



Jennings, Kaiser, Reynolds
Marine Fisheries Ecology



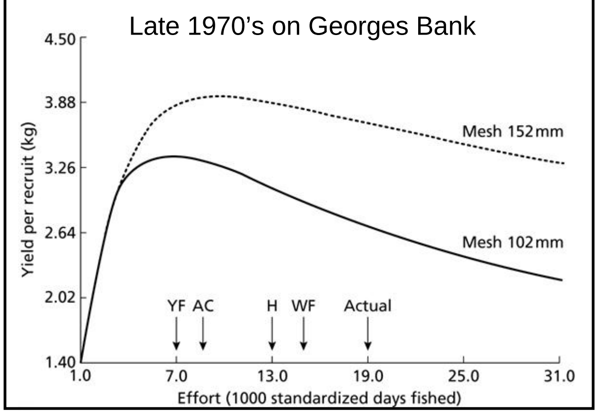
B=Belize
J=Jamaica

8.3

Virtual Population Analysis:

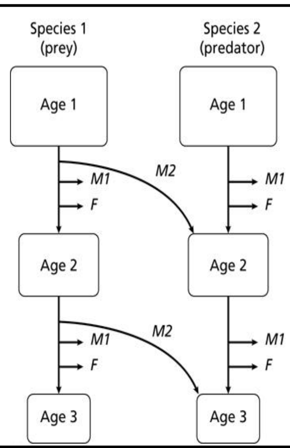
alternatives: Multispecies (Chap 8)

- Multispecies
  - Surplus production (temporal, spatial)
  - Multispecies YPR

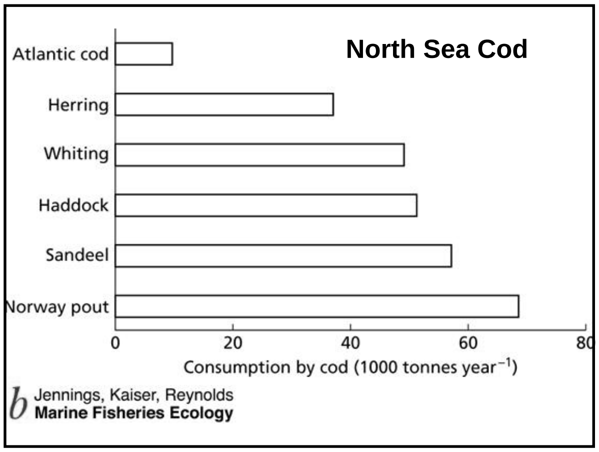


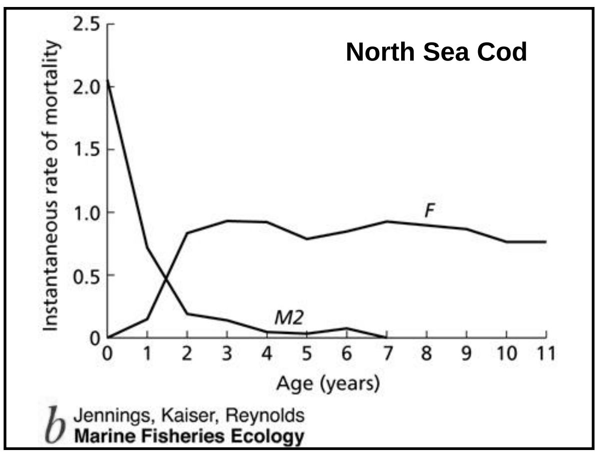
Virtual Population Analysis:
alternatives: Multispecies (Chap 8)

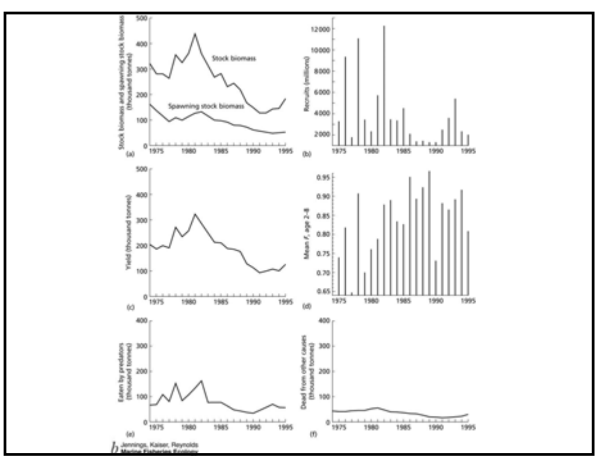
- Multispecies
  - – Surplus production (temporal, spatial)
  - – MS YPR
  - – MSVPA

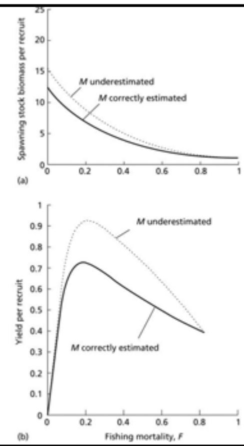


8.5


North Sea Cod
Atlantic cod
Herring
Whiting
Haddock
Sandeel
Norway pout
0
20
40
60
80
Consumption by cod (1000 tonnes year^{-1})
Jennings, Kaiser, Reynolds
Marine Fisheries Ecology


North Sea Cod
Instantaneous rate of mortality
2.5
2.0
1.5
1.0
0.5
0
F
M2
0 1 2 3 4 5 6 7 8 9 10 11
Age (years)
Jennings, Kaiser, Reynolds
Marine Fisheries Ecology

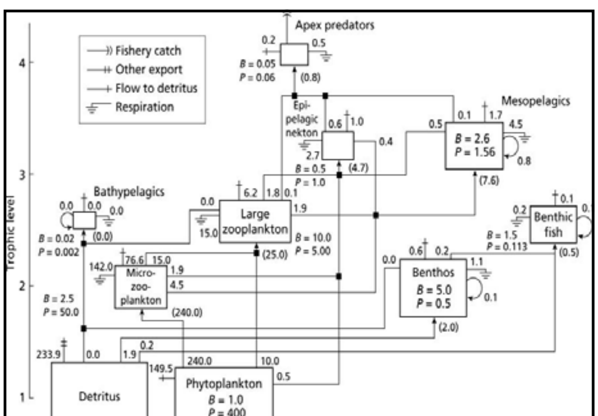

Stock biomass
Spawning stock biomass



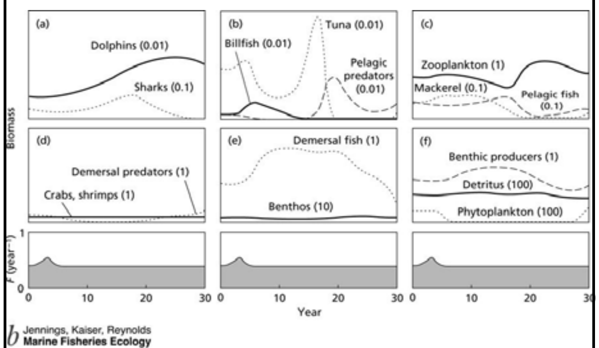
Virtual Population Analysis:

alternatives: Multispecies (Chap 8)

- Multispecies
  - Surplus production (temporal, spatial)
  - MS YPR
  - MSVPA
- Ecosystem
  - Ecopath
  - Ecosim



8.19



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Virtual Population Analysis:

Pope's approximation for VPA

Remaining fish suffer M leaving number of fish alive at the end of year equal to:

$$N_{t+1} = (N_t e^{-M/2} - C_t) e^{-M/2} \quad 7.15$$

Re-arrange to solve for N_t

$$N_t = (N_{t+1} e^{M/2} + C_t) e^{M/2} \quad 7.16$$

$$N_t = N_{t+1} e^{M} + C_t e^{M/2}$$

Virtual Population Analysis:

Pope's method:example (Box 7.4)

Step 1: identical to VPA. Find N_3 from catch equation (7.9), after estimating M and terminal F

$$N_3 = 194$$

Virtual Population Analysis:

Pope's method:example (Box 7.4)

- Step 1: identical to VPA. Find N_3 from catch equation (7.9) after estimating M and terminal F

$$N_3 = 194$$

Step 2: substitute N_3 into 7.16 as N_{t+1}

$$N_2 = (N_3 e^{M/2} + C_2)e^{M/2}$$

$$N_2 = (194e^{0.2/2} + 90)e^{0.2/2} = 336$$

Virtual Population Analysis:

Pope's method:example

Step 3: solve for F_2 from re-arranged exponential decay equation (7.7):

$$N_{t+1} = N_t e^{-(F_t + M)}$$

$$F_t = \ln\left[\frac{N_t}{N_{t+1}}\right] - M$$

$$F_2 = \ln\left[\frac{N_2}{N_3}\right] - M = \ln\left[\frac{336}{194}\right] - 0.2 = 0.349$$

Alternative cohort models:

Statistical Catch-at-age

- Based on underlying population growth model (compare model predictions to observed data)
- Catch-at-age data are analyzed one cohort at a time, i.e.…
- parameter estimates for one cohort are independent of estimates for others
  - Age-specific and year-specific F (separable)
  - Complex computationally (non-linear regression)