The Balmer Series

Introduction

Historically, the spectral lines of hydrogen have been categorized as six distinct series. The visible portion of the hydrogen spectrum is contained in the Balmer series, named after Johann Balmer who discovered an empirical relationship for calculating its wavelengths [2]. In 1885 Balmer discovered that by labeling the four visible lines of the hydorgen spectrum with integers $n, (n = 3, 4, 5, 6)$ (See Table 1 and Figure 1) each wavelength λ could be calculated from the relationship

$$\lambda = B\frac{n^2}{n^2 - 4} \,, \tag{1}$$

where $B = 364.56$ nm. Johannes Rydberg generalized Balmer's result to include all of the wavelengths of the hydrogen spectrum. The Balmer formula is more commonly re–expressed in the form of the Rydberg formula

$$\frac{1}{\lambda} = R_H \left(\frac{1}{2^2} - \frac{1}{n^2} \right) \,, \tag{2}$$

where $n = 3, 4, 5 \ldots$ and the Rydberg constant $R_H = 4/B$.

The purpose of this experiment is to determine the wavelengths of the visible spectral lines of hydrogen using a spectrometer and to calculate the Balmer constant B.

Procedure

The spectrometer used in this experiment is shown in Fig. 1. Adjust the diffraction grating so that the normal to its plane makes a small angle α to the incident beam of light. This is shown schematically in Fig. 2. Since $\alpha \approxeq 0$, the angles between the first and zeroth order intensity maxima on either side, θ and θ' respectively, are related to the wavelength λ of the incident light according to [1]

$$\lambda = d \sin(\phi) \,, \tag{3}$$

accurate to first order in α. Here, d is the separation between the slits of the grating, and

$$\phi = \frac{\theta' + \theta}{2} \tag{4}$$

Color	Wavelength [nm]	Integer [n]
Violet 2	410.2	6
Violet 1	434.0	5
blue	486.1	4
red	656.3	3

Table 1: The wavelengths and integer associations of the visible spectral lines of hydrogen shown in Fig. 3.

The angle α can be estimated by

$$\alpha = \frac{\delta \cos(\phi)}{1 - \cos(\phi)} \, , \tag{5}$$

where

$$\delta = \frac{\theta' - \theta}{2} \, . \tag{6}$$

Figure 1: Experimental setup. In the foreground is a flashlight, behind which is the spectrometer. To the right of the spectrometer is the power supply with the hydrogen discharge tube mounted accross its high voltage connectors.

The four visible spectral lines of hydrogen are shown in Fig. 3. The accepted values of their wavelengths and the integer associations assigned by Balmer are given in Table 1. The colors associated with each wavelength are based on the standard given in Fig. 4. To determine the wavelengths of these spectral lines proceed as follows.

1. Turn on the power supply to which is attached the hydrogen discharge tube.

2. Align the telescope so that the cross hairs in the eyepiece are centered on the light emerging from the collimator tube. Adjust the vernier scale so that the

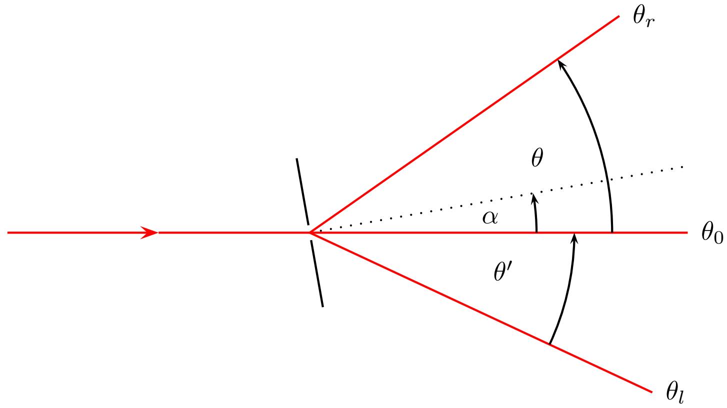

Figure 2: A depiction of the first order maximum of monochromatic light being dispersed by a diffraction grating. The angle α which is the angle the normal to the diffraction grating makes with respect to the incident beam of light is approximately zero. The quantity θ_0 is the angular coordinate of the zeroth order maximum. The quantities θ_l and θ_r are the angular coordinates of the first order maxima to the left and right of the zeroth order maximum.

angle corresponding to this location of the telescope is approximately $40°$, i.e. $\theta_0 \approx 40°$.[1]

3. Attach the diffraction grating to the platform on the spectrometer so that the normal to the plane of the grating is in the direction of the beam, i.e. set α as close to zero as possible. If the diffraction grating is positioned properly, the angles θ and θ' are approximately equal, i.e. they should not differ by more than a degree.

4. Identify the first order maxima of the four visible spectral lines to the left of the collimating tube.

5. Measure their respective angles and report them as θ_l in Table 2. Note: Because of the vernier scale angles can be measured to an accuracy of $\frac{1}{60}^{\circ}$.

6. Perform the corresponding measurements for the first order maxima to the right of the collimating tube and report their values as θ_r.

[1] The reason for this initial adjustment is to insure that all angular measurements lie between zero and 180 degrees, thereby making subsequent calculations more straightforward.

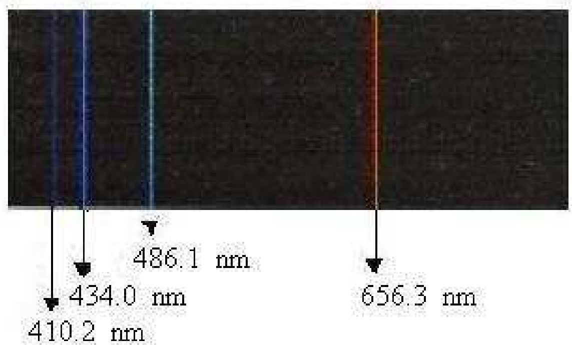

Figure 3: The four visible lines of the hydrogen spectrum are shown. These lines are denoted, from left to right, violet 2, violet 1, blue, and red.

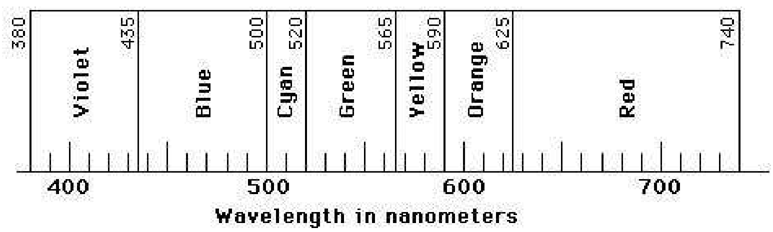

Figure 4: One standard of associating color with wavelength.

7. Calculate ϕ for each spectral line. According to Fig. 2 one can re-express Eq. 4 as

$$\phi = \frac{|\theta_l - \theta_r|}{2} \, . \tag{7}$$

Report their values in Table 2.

8. Using Eq. 3 with $d = 1.6667 \times 10^3$ nm, calculate the experimental values of the wavelengths and report them in Table 2.

9. Complete Table 2 by evaluating $\frac{n^2}{n^2-4}$ for each wavelength.

10. Using Microsoft Excel plot a graph with λ_{exp} as the ordinate and $\frac{n^2}{n^2-4}$ as the abscissa.

11. Perform a linear regression on the data, forcing the y–intercept to be zero.

12. Based on appearances and the value of r^2, i.e. the square of the correlation coefficient, does the linear regression appear to be a good fit to the data?

13. According to Eq. 1 the slope of the graph provides an estimate of the Balmer constant B whose accepted value is $B = 364.56$ nm. Report the slope in the form of Eq. 10. According to the criterion given in Eq. 11, is the experimental value of the Balmer constant, i.e. the slope, in agreement with its accepted value?

Appendix

Consider two sets of data y_i and x_i ($i = 1 \ldots N$) where the y_i are assumed to be linearly related to the x_i according to the relationship

$$y_i = mx_i + \epsilon_i \ . \tag{8}$$

The ϵ_i is a set of uncorrelated random errors. The slope m of the regression line can be estimated using a variety of techniques, such as the method of least squares estimation. It can be shown that the standard error of the slope, σ is given as

$$\sigma = \sqrt{\frac{1-r^2}{N-1}}\frac{\sigma_y}{\sigma_x} \ , \tag{9}$$

where r is the estimated correlation coefficient between the y_i and x_i, σ_y and σ_x are the standard deviations of the y_i and x_i, respectively. The quantities σ_y and σ_x can straightforwardly be calculated using function keys on a scientific calculator or defined functions in Excel. Note that if the regression equation, Eq. 8, includes a y-intercept as an additional parameter to be estimated, then the term $N-1$ in Eq. 9 is replaced by $N-2$, reflecting the extra degree of freedom in the estimation process.

To reflect its statistical uncertainty the slope is typically reported as

$$m \pm \sigma \ , \tag{10}$$

which can be understood informally to mean that with high probability the true value of the slope lies within the interval

$$[m - 1.96\sigma, m + 1.96\sigma] \ , \tag{11}$$

and its best estimate is m.

References

[1] Using the spectrometer. `http://people.uncw.edu/olszewski/phy102lab/laboratory/spectrometer.pdf`.

[2] Wikipedia. Balmer series — Wikipedia, the free encyclopedia. `http://en.wikipedia.org/wiki/Balmer_series`, 2008. [Online; accessed 2-April-2008].

	θ_l [deg]	θ_r [deg]	ϕ [deg]	n	$\frac{n^2}{n^2-4}$	λ_{exp} [nm]
Violet 2				6		
Violet 1				5		
Blue				4		
Red				3		

Table 2: Data and Calculations