Standing Waves on a String

Introduction

Consider a string, with its two ends fixed, vibrating transversely in one of its harmonic modes. See Figures 1 and 2.

Figure 1: Equipment Used in Performing the Experiment.

Locations along the string where no vibration occurs, such as the end points, are called nodes, whereas locations where maximum vibration occurs are called antinodes. The speed v with which a wave propagates along the string is given by [1]

$$v = \sqrt{\frac{T}{\mu}}\,, \tag{1}$$

where T is the tension in the string, and μ is the linear mass density. The speed is related to the wavelength λ and frequency f of the wave as follows:

$$v = f\lambda\,. \tag{2}$$

Combining these two equations we obtain

$$\lambda = \frac{1}{f\sqrt{\mu}}\sqrt{T}\,. \tag{3}$$

Figure 2: Apparatus for Generating Standing Waves on a String. A standing wave is shown with five internal nodes.

In this experiment a string is forced to vibrate at a known frequency. The tension in the string is varied so that the wavelength of waves propagating along the string also varies. A regression equation based on the form of Equation 3 is obtained using the tension and wavelength data derived from the experiment. The consistency of the theory underlying Eq. 3 is assessed by comparing the slope of the regression line to its theoretical value.

Procedure

Record all measurements and calculations to the correct number of significant figures in the appropriate table. Perform the experiment and analysis as follows.

1. Cut approximately 2.5 m of string, and tie it to the metal piece protruding from the vibrator. Tie a loop in the opposite end of the string, and drape it over the pulley. The loop of the string should be far enough from the floor so that when the mass holder is attached to the loop, it does not touch the floor.

2. Attach the mass holder to the end of the string opposite the wave generator.

3. Plug in the wave generator.

4. Add enough mass to the mass holder (total mass m) so that the string supports a standing wave with five nodes in the interior of the string.[1]

[1] If less mass than the mass of the mass holder is required to generate five nodes, use the set of weights which includes a plastic holder. The plastic holder is less massive than the metal holder. For the remainder of the experiment use the metal mass holder along with its associated masses.

5. Measure the distance L from the node at the pulley to the fifth node on the interior of the string.

6. Do the same for $n = 4, 3, 2, 1$, n being the number of nodes in the interior of the string (not including the node at the pulley). In each case, measure the distance L from the node at the pulley to the interior node closest to the vibrator.

7. Unplug the vibrator. Leave the amount of mass necessary to generate a single node still attached to the string. Mark the string with a pencil at two locations separated by one meter.

8. Now, reduce the mass to the amount of mass necessary to generate five nodes. Measure the distance between the pencil marks on the string. Retain its value for subsequent calculations.

9. Cut the string at the pencil marks. Measure the mass of the string using the jewelry balance.

10. Calculate the linear mass density of the string, μ_1, for the case when there is one internal node. Calculate the linear mass density, μ_5, for the case when there are five internal nodes. Convert the two linear mass densities to SI units, i.e. (kg/m).

11. Calculate the tension T in the string for each of the five cases. The tension is the force exerted on the string by the mass hanging from the string: $T = mg$. Here the acceleration of gravity $g = 9.80$ m/s^2.

12. Calculate the wavelength λ of the standing wave in each case, where $\lambda = \frac{2L}{n}$. Note: the distance between consecutive nodes is $\frac{\lambda}{2}$.

13. Plot a graph of λ vs $\sqrt{T}$. The quantity λ is considered the dependent variable and is plotted along the vertical axis (the ordinate).

14. Using Excel, perform a linear regression of the data, forcing the intercept to be zero.

15. Calculate the standard error in the slope using equation 8. Report the experimental value of the slope in Table 2 in the form $M_{\text{est}} \pm \delta M$, where M_{est} is the value of the slope obtained from the regression.

16. Calculate the theoretical value of the slope,M. According to Equation 3, $M = \frac{1}{f\sqrt{\mu}}$. The frequency of vibration is $f = 60$ Hz. Calculate M twice, once using μ_1 and again using μ_5. Also, calculate the 95% confidence interval using Equation 10. Report the values in Table 3.

17. Compare the theoretical and experimental values in Table 3. Are theory and experiment consistent?

Appendix

Consider two sets of data y_i and x_i ($i = 1 \dots N$) where the y_i are assumed to be linearly related to the x_i according to the relationship

$$y_i = Mx_i + B + \epsilon_i \ . \tag{4}$$

The ϵ_i is a set of uncorrelated random errors. Estimates of the slope M_{est} and y–intercept B_{est} can be obtained using a variety of techniques, such as the method of least squares estimation. The degrees of freedom $\nu = N-2$, since the number of parameters being estimated is 2, i.e. M and B. It can be shown that the standard error of the slope δM is

$$\delta M = \sqrt{\frac{1-r^2}{N-2}} \frac{\sigma_y}{\sigma_x} \ , \tag{5}$$

and the standard error of the intercept δB is

$$\delta B = \sigma_y \sqrt{\frac{1-r^2}{N-2}} \sqrt{\frac{N-1}{N} + \frac{\bar{x}^2}{\sigma_x^2}} \ , \tag{6}$$

where r is the estimated correlation coefficient between the y_i and x_i, σ_y and σ_x are the estimated standard deviations of the y_i and x_i, and $\bar{x}$ is the estimated average of the x_i.[2] The quantities $\bar{x}$, σ_x, and σ_y can straightforwardly be calculated using function keys on a scientific calculator or defined functions in Excel. If, in the regression equation, Eq. 4, the quantity B, the y-intercept, is required to be zero, i.e. the regression equation is now

$$y_i = Mx_i + \epsilon_i \ , \tag{7}$$

then the degrees of freedom $\nu = N - 1$ so that

$$\delta M = \sqrt{\frac{1-r^2}{N-1}} \frac{\sigma_y}{\sigma_x} \ . \tag{8}$$

To reflect the statistical uncertainty in a quantity Q, where Q is either the slope M or y-intercept B, the quantity Q is typically reported as

$$Q_{\text{est}} \pm \delta Q \ , \tag{9}$$

which can be understood informally to mean that, assuming the experimental results are consistent with theory, then the value of Q, predicted by theory, is likely to lie within the limits defined by Equation 9. This informal interpretation can be made more precise. Specifically, one specifies a confidence interval, e.g. the 95% confidence interval (See below.). Then assuming that the theory accounts for the experimental results, there is a 95% probability that the calculated confidence interval from some future experiment encompasses the theoretical value. If, for a given experiment the theoretical value Q lies outside of the confidence interval, the assumption that theory

[2] Note: in Equation 6 σ_x and σ_y are unbiased estimates.

accounts for the results of the experiment is rejected, i.e. the experimental results are inconsistent with theory. The confidence interval is expressed as

$$[Q_{\text{est}} - X\ \delta Q, Q_{\text{est}} + X\ \delta Q]\ , \tag{10}$$

The quantity X is obtained from a Student's t-distribution and depends on the confidence interval and the degrees of freedom. A detailed and illuminating discussion of the Student's t-distribution can be found in the Wikipedia on-line free encyclopedia. [2] There are various ways of obtaining or calculating the value of X. For example, the spreadsheet Microsoft Excel includes a library function T.INV.2T for calculating X based on a two-tailed t-test. Specifically,

$$X = \text{T.INV.2T}(p, \nu)\ , \tag{11}$$

where the probability $p = 1 - \frac{\text{(the confidence interval)}}{100}$ and ν is the degrees of freedom. Consider the following example for illustrative purposes. The number of data points is 5; the confidence interval is 95%. The data are assumed to be linearly related with an intercept of zero so that Equation 7 applies. Therefore

$$X = \text{T.INV.2T}(1 - \frac{95}{100}, 5 - 1) = 2.776\ , \tag{12}$$

References

[1] Wikipedia. Vibrating string. `http://en.wikipedia.org/wiki/Vibrating_string`, 2008. [Online; accessed 12-June-2008].

[2] Wikipedia. Student's t-distribution. `https://en.wikipedia.org/wiki/Student%27s_t-distribution`, 2017. [Online; accessed 22-March-2017].

Number of nodes interior to the string	m (kg)	T (nt)	λ (m)	$\sqrt{T(\text{nt})}$
1				
2				
3				
4				
5				

Table 1: Data and Calculations I

	Experiment
Slope	$\pm$

Table 2: Calculations II

$Q_{\text{est}} - X\ \delta Q$	$Q_{\text{est}} + X\ \delta Q$	Theory (1 node)	Theory (5 node)
$-$	$+$		

Table 3: Calculations III