

# Rotational Equilibrium of a Rigid Body

## Introduction

Consider a rigid body acted upon by  $N$  forces. The two conditions which must be satisfied in order that the rigid body be in both translational and rotational equilibrium are,

$$\sum_{i=1}^N \vec{F}_i = 0 , \quad (1)$$

and

$$\sum_{i=1}^N \vec{\tau}_i = 0 . \quad (2)$$

Here, the  $\vec{F}_i$  and  $\vec{\tau}_i$  are the individual forces and corresponding torques, about some pivot point, acting on the rigid body (See reference [1] and Eq. 3 in Appendix A.).

In Cases I and II of the experiment a rigid body (a meter stick) is subjected to various combinations of forces in such a way that the body remains in equilibrium. In each case the forces and torques exerted on the rigid body are determined from data collected in the experiment. The calculated value of the net torque derived from the experiment is compared for consistency with the theoretical value of zero predicted by Eq. 2. In Case III of the experiment, Eq. 2 is applied in a novel way to predict the mass of an object. The predicted value is then compared for accuracy to the value obtained from a triple beam balance.

## Procedure

The equipment used in performing this experiment is shown in Figure 1. Record all measurements and calculations in the appropriate table.

1. Measure the mass of the meter stick.
2. Measure the mass of the metallic object.
3. Determine the center of mass of the meter stick by balancing the meter stick on the knife edge.

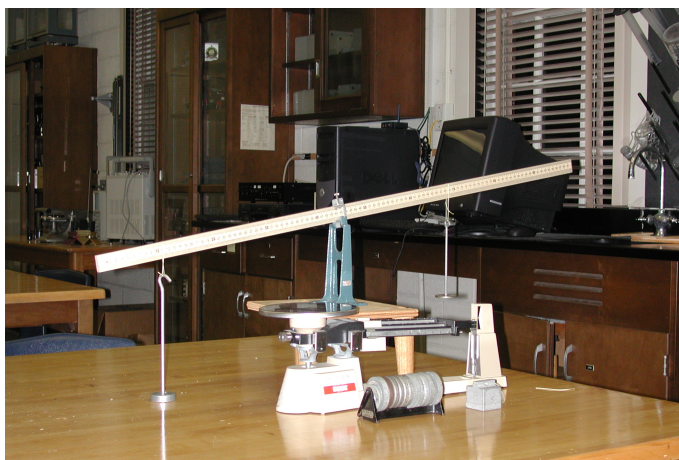


Figure 1: Torque Apparatus

### Case I

Determine the pivot point of the meter stick when 150 grams of mass are suspended from the 20 cm mark of the meter stick. Report its position in Table 2.

### Case II

Determine the pivot point of the meter stick when 150 grams of mass are suspended from the 20 cm mark and 250 grams are suspended from the 70 cm mark of the meter stick. Report its position in Table 2.

### Case III

Suspend the metallic object from the 10 cm mark of the meter stick. Suspend 250 grams from the meter stick so that the masses and the meter stick balance at the center of mass of the meter stick. Report its position in Table 3.

## Analysis

1. For both Case I and Case II calculate the counterclockwise torque, the clockwise torque, the net torque, and the percent discrepancy, as given by Eq. 5 in Appendix B.
2. For Case III calculate the theoretical value of the metallic mass, using Eq. 2. Calculate the percent error, as given by Eq. 4 in Appendix B.

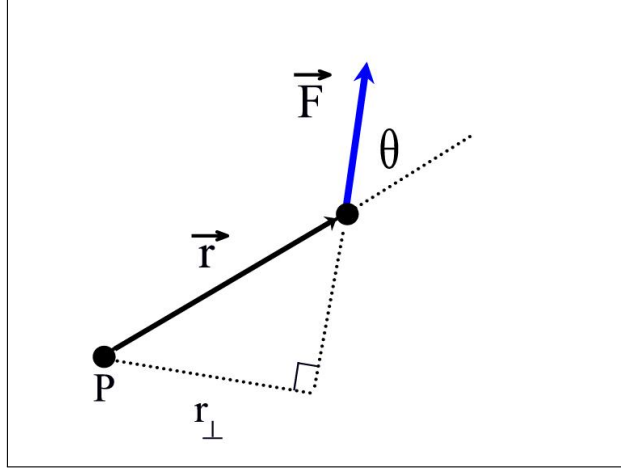


Figure 2: The Definition of Torque

## Appendix A

In general, the magnitude of the torque  $\tau$  resulting from a force  $\vec{F}$  about some pivot point  $P$  is defined as

$$\tau = |\vec{r}| |\vec{F}| \sin(\theta) = r_{\perp} |\vec{F}|. \quad (3)$$

The quantity  $r_{\perp}$  is the length of the line connecting the pivot point, perpendicularly, to the line of action of the force, as shown in Figure 2. The torque is assigned a positive (negative) value if it can be associated with a counterclockwise (clockwise) rotation.

## Appendix B

The percent error  $p_e$  in the mass is defined as:

$$p_e = 100 \times \frac{|m_{\text{exp}} - m_{\text{theo}}|}{m_{\text{exp}}}, \quad (4)$$

where  $m_{\text{exp}}$  and  $m_{\text{theo}}$  are the measured and theoretical values of the mass.

The percent discrepancy  $p_d$  in the torque is defined as

$$p_d = 100 \times \frac{|\tau_{\text{net}}|}{(\sum_i |\tau_i|)/2}, \quad (5)$$

where  $\tau_{\text{net}}$  is the net torque, and the  $\tau_i$  include both clockwise and counterclockwise torques.

## References

- [1] Wikipedia. Torque. <http://en.wikipedia.org/wiki/Torque>, 2007.  
[Online; accessed 19-October-2007].

Mass of Meter Stick (kg)	Mass of Metallic object (kg)	Center of Mass of the Meter Stick (m)

Table 1: Data

	Position of Knife Edge (m)
Case 1	
Case 2	

Table 2: Data

	Position of 250 g mass (m)
Case 3	

Table 3: Data

	Case I	Case II
CC Torque (Nt-m)		
C Torque (Nt-m)		
Net Torque (Nt-m)		
Percent Discrepancy		

Table 4: Calculations

	Case III
Mass (theoretical) (kg)	
Percent Error	

Table 5: Calculations