Vector Algebra

Introduction

Vectors are used in physics because they provide a concise conceptual framework for characterizing a number of physical quantities, e.g. displacement, velocity, force, etc. A practical approach to learning the rudiments of vector algebra is by the direct application of vector algebra to the analysis of an experiment involving forces in equilibrium. Consider an object which is acted upon by N forces, $\vec{F}_i (i = 1 \ldots N)$. According to Newton's Second Law, in order for the object to be in static equilibrium the net force on the object must be zero. Thus, the additional force $\vec{X}$ required to place the object in static equilibrium satisfies the relationship

$$\vec{X} + \sum_{i=1}^{N} \vec{F}_i = 0 \,, \tag{1}$$

so that

$$\vec{X} = -\sum_{i=1}^{N} \vec{F}_i \,. \tag{2}$$

Figure 1: Equipment

Case	$\vec{F}_1$ (gw)	$\vec{F}_2$ (gw)	$\vec{F}_3$ (gw)	$\vec{X}_{\text{exp}}$ (gw)	$\vec{X}_{\text{anal}}$ (gw)	$\vec{X}_{\text{graph}}$ (gw)	SE (gw)
1	$100@0°$	$150@120°$					
2	$150@20°$	$200@110°$	$100@225°$				

Table 1: Data and Results

Procedure

To simplify the ensuing analysis we introduce an unconventional unit of force, the gram weight. One gram weight (gw) is defined as the weight of an object whose mass is one gram.

Experiment

Case 1

1. On the force table attach 100 gw to one string at an angle of $0°$. Attach 150 gw to another string at an angle of $120°$.
2. Attach to the third string at the appropriate angle the amount of weight necessary to restore the ring to static equilibrium, i.e. so the net force exerted on the ring is zero. In Table 1 report the force $\vec{X}_{\text{exp}}$, expressed as a magnitude and a direction.

Case 2

1. Repeating step one of the previous case attach 150 gw at an angle of $20°$, 200 gw at an angle of $110°$, and 100 gw at an angle of $225°$.
2. Determine the force $\vec{X}_{\text{exp}}$ to restore the ring to static equilibrium. Report the result in Table 1.

Analysis

1. For each case solve Eq. 2 using a graphical technique. Report the results in Table 1 as $\vec{X}_{\text{graph}}$.
2. For each case solve Eq. 2 analytically. Report the results in Table 1 as $\vec{X}_{\text{anal}}$.[1] In addition, calculate, analytically, the standard error (SE) in the measurement, which is the magnitude of the difference between the vectors $\vec{X}_{\text{exp}}$ and $\vec{X}_{\text{anal}}$, i.e. SE $\equiv |\vec{X}_{\text{exp}} - \vec{X}_{\text{anal}}|$.

[1] In applying the analytical technique to case two, a judicious choice of the coordinate system can reduce the number of computations.