Measurement

Introduction

With few exceptions the basis of physical inquiry is experiment. An experiment typically involves collecting numerical data which may be used in testing the validity of a physical theory. Usually, experimental data are corrupted by the occurrence of random errors which the experimenter is unable to eliminate. Thus, although there may be a unique value for some physical measurement, it may be impossible to determine its value within the measuring accuracy of the apparatus used in the experiment. In such cases, only a best estimate of the physical measurement can be given, along with some measure of confidence in its value. Two quantities used in this regard are the mean, $\bar{x}$, and the standard deviation of the mean, σ, which can be estimated as follows:

$$\bar{x} = \frac{\sum_{i=1}^{N} x_i}{N} , \tag{1}$$

where N is the number of measurements, and x_i is one of the measurements. The quantity

$$\sigma = \frac{\sigma_{N-1}}{\sqrt{N}} , \tag{2}$$

where an unbiased estimate of the the standard deviation of the population of measurements is given by

$$\sigma_{N-1} = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N-1}} . \tag{3}$$

Qualitatively, $\bar{x}$ represents the best estimate of the true value of the measurement, and σ represents the confidence in that value. The physical measurement is reported in the form

$$\bar{x} \pm \sigma . \tag{4}$$

In some experiments it may be hypothesized that a relationship exists among several different types of measurements. In the simplest case, there are two types which are related linearly. If the measurements include random errors, it is not possible to determine, definitively, whether or not a linear relationship exists between the different types of measurements. Nonetheless, using a technique called regression analysis, it is possible to obtain a best linear fit to the data and a measure of confidence in the fit. Using a statistical technique which minimizes the deviation of the data from the regression line, it is possible to estimate the parameters associated with the regression line, i.e. slope and intercept. It is also possible to obtain a measure of confidence in the fit , R^2 ($0 \leq R^2 \leq 1$). The quantity R is called the correlation coefficient. Values of R^2 near zero suggest the least confidence in the fit. i.e. that the variation in values of one of the data series is not adequately explained by the hypothesized linear relationship between the different types of measurements, and values near one suggest the most confidence in the fit.

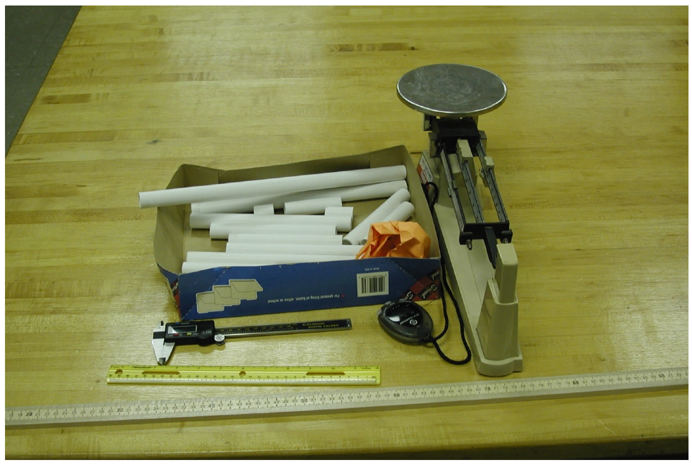

Figure 1: Equipment

Procedure

There are two experiments to be performed. In the first experiment a crumpled piece of paper is dropped from the same height on five occasions, and its time of flight is measured on each occasion. The best estimate of the time of flight and the confidence in its value is reported. In the second experiment the length and mass of five pieces of plastic pipe are measured. A graph of length vs. mass is plotted. Then, length is regressed, linearly, on mass under the assumption that the intercept of the regression line is zero. The equipment used in this experiment is shown in Figure 1.

Time of Flight of a Crumpled Piece of Paper

1. Select a convenient location from which to drop the crumpled paper. Measure the distance from the floor to the location in centimeters.

2. Drop the crumpled paper, measuring its time of flight. Perform this experiment four additional times and record the measurements in Table 1.

3. Calculate $\bar{t}$ and σ, and report the time of flight, as described in Equation 4.

4. Calculate the theoretical value for the time of flight using the formula $t_{\text{theo}} = \sqrt{H/490}$ where H is the distance to the floor. Report the value in Table 1.

5. Are the results of the experiment consistent with the theoretical value for the time of flight?

The Relationship between Mass and Length of Pieces of Plastic Pipe

1. Measure the mass and length of five pieces of plastic pipe. Choose pieces whose lengths are representative of the entire range of lengths. Record the data in Table 2.

2. By hand, plot a graph of length vs. mass, and estimate a linear fit to the data, whose y-intercept is zero (Length and mass are plotted as the ordinate and abscissa, respectively.). Calculate the slope of the regression line.

3. Using Microsoft Excel, plot a graph of the data, and perform a linear regression, forcing the intercept of the regression line to be zero. Report the slope and the value of R^2 in Table 2.

4. Compare the slopes obtained from the two graphs. They should be approximately equal.

5. Does the regression line provide an adequate fit to the data? If so, why would you expect this to be the case?

Trial	t (s)	H (cm)
1		
2		
3		
4		
5		
Time ($\bar{t} \pm \sigma$)		
Time t_{theo}		

Table 1: Time of Flight Data and Calculations

Trial	m (gm)	l (cm)
1		
2		
3		
4		
5		
Slope		
R^2		

Table 2: Mass and Length Data and Calculations