A Precise Determination of the Acceleration of Gravity

Introduction

Consider an object moving in one dimension. Assume that the object is located initially at position $x_0 = 0$ and moving with velocity v_0. Further, assume that the object is undergoing constant acceleration, a. Then the following kinematical equation applies:

$$v^2 = v_0^2 + 2ax \; , \tag{1}$$

where x is the displacement of the object from its initial position, and v is the instantaneous velocity of the object at the displacement x. In addition, the average velocity $\bar{v}$ of an object which has undergone a displacement Δx during a time interval Δt is

$$\bar{v} = \frac{\Delta x}{\Delta t} \; . \tag{2}$$

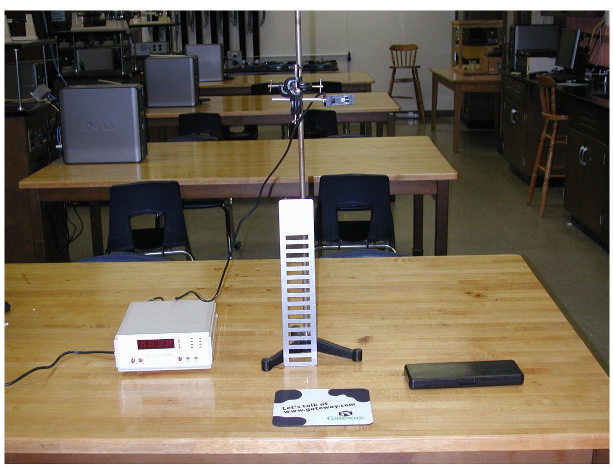

Figure 1: Equipment

Procedure

Near the surface of the earth the acceleration due to gravity is approximately $980\text{cm}/s^2$. The purpose of this experiment is to obtain a precise value for the acceleration of gravity in the laboratory. Its methodology includes dropping a metal plate and collecting data which are used to obtain approximate values for the instantaneous velocity of the plate at various displacements during its flight. The velocity and displacement values are then fit to Eq. 1 with the values a and v_0 determined to optimize the fit. The value of R^2 is obtained to assess confidence in the fit, and the experimental value of the acceleration of gravity is obtained from the fit. Then, the acceleration of gravity in the laboratory is calculated using a semi-empirical formula. Finally, the two values are compared for consistency.

The metal plate shown in figure 2 comprises a series of one centimeter pieces of solid metal alternating with one centimeter openings. It is dropped from several centimeters above the photogate so that it interrupts the beam of light. The timer begins recording time when the first piece of metal interrupts the beam and stops when the next consecutive piece of metal interrupts the beam. Thus, the timer records the amount of time, Δt, for the metal piece to be displaced by an amount $\Delta x = 2.00$ cm. The timer then remains inactive until the next piece of metal enters the beam. Thus, a series of time measurements is obtained for two centimeter displacements (one centimeter of metal and one centimeter of opening) of the metal plate. (The midpoint of each consecutive displacement is separated by 4.00 cm.).

Turn on the timer. Set the timer to function S2. Drop the metal plate, as described. After the metal plate has completed its flight, stop the timer, and obtain the time measurements.

1. Measure the width of several metal strips and alternating openings of the plate to verify that the width of each strip and opening is 1.00 cm to three significant figures.

2. Using Eq. 2 calculate the average velocity, $\bar{v}$ during each time interval, Δt, recorded by the timer.

3. Using Excel plot a graph of $\bar{v}^2$ vs. x. Here x is the displacement of the plate where its instantaneous velocity equals the average velocity $\bar{v}$ of each displacement of two centimeters. Assume that the displacement x where this occurs is $x =$1.00 cm, 5.00 cm, 9.00 cm $\ldots$. [1] Perform a linear regression, reporting the estimated regression parameters in Eq. 1 and the value of R^2. If $R^2 = 1$, then the value of g estimated is exact to the accuracy of the measurements, i.e. there is no random error in the estimated value of g.

[1] This assumption, which simplifies the analysis, in fact, is not correct. The location where the average velocity occurs is given by $\frac{d}{2} - \frac{1}{8}g\Delta t^2$, where $d = 2$cm, g is the acceleration of gravity, and Δt is the time to traverse the 2 cm. It can be shown that the systematic error resulting from this assumption occurs in the third significant figure of g. It is interesting to note that neglecting the effect of air resistance results in a systematic error in the fourth or fifth significant figure.

4. Calculate the theoretical value of the acceleration of gravity in the lab, g_{theory}, using the Clairaut formula with the free air correction (This formula is discussed in detail, online, in the Wikipedia encyclopedia, Standard gravity.):

   $$g_{\text{theory}} = 978.0327[1+5.3024\times10^{-3}\sin^2(L)-5.8\times10^{-6}\sin^2(2L)]-3.086\times10^{-6}H.$$

   Here, the latitude of the lab is L, ($L = 34.23°$) and its height above sea level is H, ($H = 1.3 \times 10^3$ cm.).[2]

5. Is the experimental value of g in agreement with its theoretical value? In principle, if $R^2 = 1$, the experimental value should equal the theoretical value to three significant figures. If $R^2 = 1$ and the experimental and theoretical values of g are not in agreement, offer suggestions for the disagreement between theory and experiment (See footnote 1.).

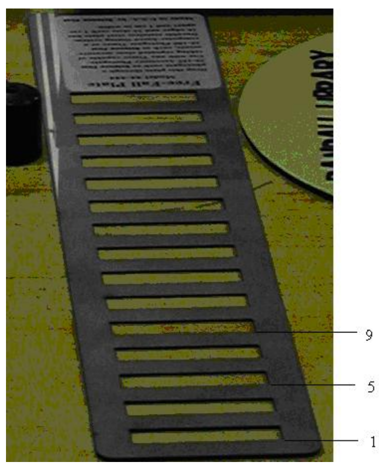

Figure 2: Metal Plate

[2]Latitude and elevation have been provided by Dr. Brian Davis.

Δt (s)	Δx (cm)	$\bar{v}$ (cm/s)	x (cm)	$\bar{v}^2$ (cm/s)^2
	2.00		1.00	
	2.00		5.00	
	2.00		9.00	
	2.00		13.00	
	2.00		17.00	
	2.00		21.00	
	2.00		25.00	
	2.00		29.00	

Table 1: Data and Calculations