Measuring the Coefficient of Kinetic Friction

Introduction

When two surfaces in contact move relative to one another in a direction tangent to the surfaces, each surface exerts a shearing force on the other. This force referred to as kinetic friction, although complicated in origin, satisfies a simple mathematical relationship,

$$f_k = \mu_k N \, , \tag{1}$$

where f_k, the kinetic friction force, is exerted tangent to the surface and opposite to its direction of motion. Here, N is the force that one surface exerts normal to the other surface, and μ_k, the coefficient of kinetic friction, depends only on the intrinsic properties of the two surfaces in contact. [1]

The purpose of this experiment is to determine the coefficient of kinetic friction between the bottom surface of a wooden box, lined with felt, and a glass plate positioned on an inclined plane, as shown in Figure 1. Given the block positioned on the plane

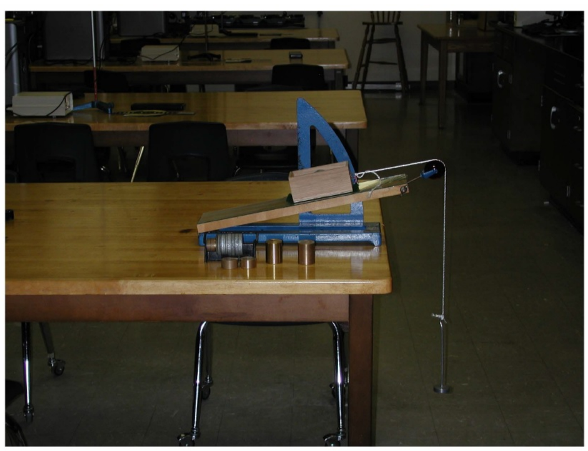

Figure 1: Box and Inclined Plane

inclined at angle θ, let M be the amount of mass suspended from the string, such that when the box, initially at rest, is nudged up the incline, it moves with constant velocity. Correspondingly, let m be the amount of mass attached to the string, such that when the box is nudged down the incline, it moves with constant velocity down the incline. It can be shown that

$$\frac{M-m}{M+m} = \mu_k \cot(\theta)\ . \tag{2}$$

Procedure

Insert a 500 gram mass and two 100 gram masses in the box, as shown in Fig. 2.

1. Without the mass holder attached, determine the angle of elevation of the plane such that when the box is nudged down the incline, it moves with constant velocity (See Appendix A.). Since there is no mass attached to the string, this corresponds to $m = 0$ described in the introduction. Attach the mass holder to the string, and add a sufficient amount of mass so that when the box is nudged up the incline, it moves with constant velocity, as described in the introduction.[1] Record these data in Table 1 as Case one.

2. Raise the plane approximately $5°$ above the angle obtained in step one, and determine M and m, as described in the introduction. Report these values in Table 1, as Case two.

3. Do the same for three additional, approximately, $5°$ increments, and report the values in Table 1 accordingly. Do not exceed an angle of elevation of $45°$.

4. Plot a graph of $\cot(\theta)$ vs. $\frac{M-m}{M+m}$, where the former is the abscissa and the latter is the ordinate. Note: since case one corresponds to the situation when $m = 0$, $\frac{M-m}{M+m}$ evaluates to one independent of the value of M.

5. Perform a linear regression of the data, forcing the intercept to be zero. Report the values of the estimated regression parameters and the value of R^2.

6. Calculate the standard error, δM, of the slope using equation 7.

7. According to Eq. 2, μ_k is equal to the slope of the regression line. In Table 2 report μ_k, i.e. report the experimental value of the slope in the form $M_{\text{est}} \pm \delta M$, where M_{est} is the slope of the regression line.

A Appendix

In performing the experiment you are instructed to nudge the box so that it moves with constant velocity. In order to obtain acceptable experimental results, this procedure should be performed in a consistent way. The following is a suggestive, but not necessarily prescriptive, way of satisfying this condition:

[1] When adding mass to the mass holder, do so in increments of 20 grams or larger throughout the experiment.

Figure 2: Box and Weights. The 500 gram and two 100 gram masses are placed in the box as shown.

1. Position the box at suitable location on the plane. This location should be used throughout the experiment since the intrinsic properties of the plane on which the coefficient of kinetic friction depends are, in general, not homogeneous along the surface of the plane;

2. Press the box gently against the plane to insure that it is making good contact with the plane;

3. Give the box an abrupt, but not excessively hard push in the direction in which it is desired that the box should move (The box should *not* acquire a large initial speed.). If, after the initial push, the speed of the box slows quickly to the point where the box creeps along for an additional distance of approximately 6 cm or more, then the condition of constant velocity can be assumed satisfied.

B Appendix

Consider two sets of data y_i and x_i $(i = 1 \ldots N)$ where the y_i are assumed to be linearly related to the x_i according to the relationship

$$y_i = M x_i + B + \epsilon_i \; . \tag{3}$$

The ϵ_i is a set of uncorrelated random errors. Estimates of the slope M_{est} and y–intercept B_{est} can be obtained using a variety of techniques, such as the method of least squares estimation. Furthermore, it can be shown that the standard error of the slope δM is

$$\delta M = \sqrt{\frac{1 - r^2}{N - 2}} \, \frac{\sigma_y}{\sigma_x} \; , \tag{4}$$

and the standard error of the intercept δB is

$$\delta B = \sigma_y \sqrt{\frac{1-r^2}{N-2}} \sqrt{\frac{N-1}{N} + \frac{\bar{x}^2}{\sigma_x^2}} \; , \tag{5}$$

where r is the estimated correlation coefficient between the y_i and x_i, σ_y and σ_x are the estimated standard deviations of the y_i and x_i, and $\bar{x}$ is the estimated average of the x_i.[2] The quantities $\bar{x}$, σ_x, and σ_y can straightforwardly be calculated using function keys on a scientific calculator or defined functions in Excel. If, in the regression equation, Eq. 3, the quantity B, the y-intercept, is required to be zero, i.e. the regression equation is now

$$y_i = Mx_i + \epsilon_i \; , \tag{6}$$

then the degrees of freedom in the random errors are $N-1$ rather than $N-2$ so that

$$\delta M = \sqrt{\frac{1-r^2}{N-1}} \, \frac{\sigma_y}{\sigma_x} \; . \tag{7}$$

To reflect the statistical uncertainty in a quantity Q, where Q is either the slope M or y-intercept B, the quantity Q is typically reported as

$$Q_{\text{est}} \pm \delta Q \; , \tag{8}$$

which can be understood informally to mean that, assuming the experimental results are consistent with theory, then the value of Q, predicted by theory, is likely to lie within the limits defined by Equation 8.

References

[1] Wikipedia. Friction. `http://en.wikipedia.org/wiki/Friction`, 2012. [Online; accessed 21-March-2012].

[2]Note: in Equation 5 σ_x and σ_y are unbiased estimates.

Case	θ°	M (gms)	m (gms)	$\cot(\theta)$	$\frac{M-m}{M+m}$
1					
2					
3					
4					
5					

Table 1: Data and Computations

$\pm$

Table 2: The experimental value of the coefficient of kinetic friction, μ_k