Magnetic Monopoles: from Dirac to D-branes

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Outline

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Electromagnetism
- The Maxwell Theory
- Compact Notation
- Minimal Coupling (Quantum Mechanics)

The Dirac Monopole

Magnetic Monopoles and Dyons in Grand Unified Theories

Montonen–Olive Duality

Superstring Theory Primer

Magnetic Monopoles in String Theory

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The Maxwell Equations
(Lorentz Heaviside units, Minkowski metric, i.e. $g_{tt} = -1$, $\hbar = 1$, $c = 1$)

$$\nabla \cdot \mathbf{E} = \rho_e \qquad \nabla \times \mathbf{B} - \frac{\partial \mathbf{E}}{\partial \mathbf{t}} = \mathbf{J_e}$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial \mathbf{t}} = 0$$

Maxwell - magnetic   Potential

The Lorentz Force Equation

$$\mathbf{F} = q_e \mathbf{E} + q_e \mathbf{v} \times \mathbf{B}$$

The Maxwell Equations (magnetic)

$$\mathbf{E} \to \mathbf{B} \qquad \mathbf{B} \to -\mathbf{E}$$
$$\mathbf{J}_e \to \mathbf{J}_m \qquad \mathbf{J}_m \to -\mathbf{J}_e$$
$$\rho_e \to \rho_m \qquad \rho_m \to -\rho_e$$

$$\nabla \cdot \mathbf{B} = \rho_m \qquad -\nabla \times \mathbf{E} - \frac{\partial \mathbf{B}}{\partial t} = \mathbf{J}_m$$
$$-\nabla \cdot \mathbf{E} = 0 \qquad \nabla \times \mathbf{B} - \frac{\partial \mathbf{E}}{\partial t} = 0$$

Maxwell - electric

The Lorentz Force Equation

$$\mathbf{F} = q_m \mathbf{B} - q_m \mathbf{v} \times \mathbf{E}$$

The Potential Function

Consequently,

$$\nabla \times \mathbf{A} = \mathbf{B} \qquad \text{since} \qquad \nabla \cdot (\nabla \times \mathbf{A}) = 0$$

$$-\nabla A^0 = \mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} \qquad \text{since} \qquad \nabla \times \nabla A^0 = 0$$

or

$$-\nabla A^0 - \frac{\partial \mathbf{A}}{\partial t} = \mathbf{E}\,.$$

▸ Maxwell - electric

Gauge Transformation

E and **B** are invariant under

$$A^0 \rightarrow A^0 + \frac{\partial \chi}{\partial t} \qquad \mathbf{A} \rightarrow \mathbf{A} - \nabla \chi$$

Notation: $(\mu = 0, 1, 2, 3; i = 1, 2, 3)$, (Minkowski metric: $g_{00} = g_{tt} = -1$), (Levi-Cevità symbol: $\epsilon_{0123} = \epsilon_{123} = 1$)

Gauge Potential: $A = A_\mu dx^\mu = A_0 dt + A_i dx^i$

Gauge Transform: $A \to A - d\chi(x^\mu)$

Field Strength: $F = -dA$

(gauge invarient $= \frac{1}{2}(\partial_\mu A_\nu - \partial_\nu A_\mu) dx^\mu \wedge dx^\nu$

due to $dd\chi = 0$) $= \frac{1}{2} F_{\mu\nu} dx^\mu \wedge dx^\nu$

E and **B**

$$F = E^k \delta_{ki} dx^0 \wedge dx^i + \frac{1}{2} B^k \epsilon_{kij} dx^i \wedge dx^j$$

$$^*F \equiv F_{\alpha\beta}\, \epsilon^{\alpha\beta}_{\mu\nu}\, dx^\mu \wedge dx^\nu$$

$$= B^k \delta_{ki} dx^0 \wedge dx^i - \frac{1}{2} E^k \epsilon_{kij} dx^i \wedge dx^j$$

duality : $F \to^* F, \quad (E^i \to B^i,\ B^i \to -E^i)$

$$^{**}F = -F$$

Euler equation (inhomogeneous): $\delta F = -{}^*d^*F = j$

$$\begin{aligned}\delta F =& -\nabla \cdot \mathbf{E}\, dt + \\ &+ (-\partial_t \mathbf{E} + \nabla \times \mathbf{B}) \cdot d\mathbf{x} \\ j_e =& j_{e\mu}\, dx^\mu = -\rho_e dt + \mathbf{J_e} \cdot d\mathbf{x}\end{aligned}$$

Bianchi identity (homogeneous): $dF = -ddA = 0$

$$\begin{aligned}dF =& \nabla \cdot \mathbf{B} dx^1 \wedge dx^2 \wedge dx^3 + \\ &+ \frac{1}{2}(\partial_t \mathbf{B} + \nabla \times \mathbf{E})^i \epsilon_{ijk} dt \wedge dx^i \wedge dx^j \\ =& 0\end{aligned}$$

Lagrangian density: $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - j_e^\mu A_\mu$

Minimal Coupling

$$p_\mu = -i\partial_\mu \to -i(\partial_\mu - ig_e A_\mu)\ ,$$

where g_e is the electric charge.

Schrödinger equation: $(\mathbf{A}, V, \chi$ time independent)

$$i\partial_t\psi = -\frac{1}{2m}(\nabla - ig_e\mathbf{A})^2\psi + g_e V\psi$$

Compactify the range of χ, i.e. χ and $\chi + \frac{2\pi}{e}$ correspond to the same gauge transformation ($e \equiv$ coupling constant).

Gauge Transformation:

$$\psi \to e^{-ig_e\chi}\psi$$

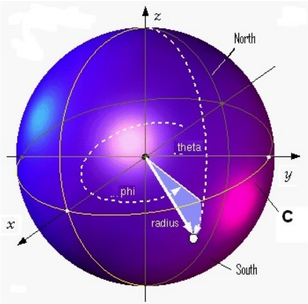
$$\mathbf{A} \to \mathbf{A} - \nabla\chi \equiv$$

$$\equiv \mathbf{A} - \frac{i}{g_e} e^{ig_e\chi}\nabla e^{-ig_e\chi}$$

Wu-Yang construction

$$A_{\substack{N\\S}} = (\pm\frac{C}{2} - \frac{g_m}{4\pi}\cos\theta)\,\frac{\hat{e}_\phi}{r\sin\theta}$$
$$= \pm\frac{C}{2}d\phi - \frac{g_m}{4\pi}\cos\theta\, d\phi$$

$$F = -dA \rightarrow \mathbf{B} = \frac{g_m\hat{\mathbf{r}}}{4\pi r^2}$$



Electric Charge Quantization

Since

$$e^{\left(-ig_e \int_0^{2\pi/e} d\chi\right)} = 1 \,,$$

then

$$g_e \frac{2\pi}{e} = 2\pi n_e \qquad\qquad n_e \in Z$$

$$g_e = n_e \, e$$

Magnetic Charge Quantization

To remove the string singularity set

$$\frac{g_m}{4\pi} = \frac{C}{2}\,.$$

Since

$$d\chi = -(A_N - A_S) = -Cd\phi = -\frac{g_m}{2\pi}d\phi$$

$$e^{\left(-ie\int_0^{2\pi}\frac{d\chi}{d\phi}\,d\phi\right)} = e^{ieg_m} = 1$$

$$g_m = n_m\frac{2\pi}{e}, \qquad n_m \in Z$$

$g_e g_m = 2\pi$ Dirac monopole ($n_m = n_e = 1$)

Summary

- The existence of a single magnetic charge requires that electric charge be quantized.
- The quantities $e^{(-ig_e\chi)}$ are elements of a $U(1)$ group of gauge transformations. If electric charge is quantized, then $\chi = 0$ and $\chi = 2\pi/e$ (where e is the coupling constant) yield the same gauge transformation, i.e. the range of χ is compact. In this case the gauge group is the circle group $U(1)$. In the alternative case when charge is not quantized and the range of χ is not compact, i.e $e \to 0$, the gauge group is the real line R^1. Magnetic monopoles require a compact $U(1)$ gauge group.
- Mathematically, we have constructed a non-trivial principal fiber bundle with base manifold S^2 and fiber $U(1)$, for the case $n_m = 1$.

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Fibre Bundles

- The gauge group of electromagnetism is the real numbers, R, under the group operation of addition or its compact equivalent the circle, $U(1)$. Attach a copy of this group to each point of spacetime. This mathematical structure is a principal fiber bundle.
- The gauge groups corresponding to other forces in nature are obtained by substituting for the gauge group of electromagnetism another appropriate Lie group.
- For the remainder of this presentation the focus will be on the gauge groups $SU(N)$ and $G2$, which may be relevant to Grand Unification, e.g. $SU(3) \times SU(2) \times U(1)$ is a subgroup of $SU(5)$; the non-compact version of $E8$ breaks down to $SU(3)$ through $G2$.

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The Yang–Mills–Higgs Lagrangian

$$\mathcal{L} = -\frac{1}{4}\boldsymbol{F}_{\mu\nu} \cdot \boldsymbol{F}^{\mu\nu} + \frac{1}{2} D_\mu \boldsymbol{\Phi} \cdot D^\mu \boldsymbol{\Phi} - V(\boldsymbol{\Phi} \cdot \boldsymbol{\Phi}) \,,$$

where

$$\boldsymbol{F}_{\mu\nu} = \partial_\mu \boldsymbol{A}_\nu - \partial_\nu \boldsymbol{A}_\mu - ie\, \boldsymbol{A}_\mu \wedge \boldsymbol{A}_\nu \,.$$

Higgs field $\boldsymbol{\Phi}$ - scalar transforming in the adjoint representation of the gauge group so that

$$D_\mu \boldsymbol{\Phi} = \partial_\mu \boldsymbol{\Phi} - ie\, \boldsymbol{A}_\mu \wedge \boldsymbol{\Phi} \,.$$

$$\boldsymbol{F}_{\mu\nu} = F^a_{\mu\nu} T_a$$

$$\boldsymbol{A}_\nu = A^a_\mu T_a$$

$$\boldsymbol{\Phi} = \Phi^a T_a$$

Note: $\boldsymbol{H} \cdot \boldsymbol{G} \equiv H^a G^b \, 2\mathrm{Tr}(T_a T_b) = H^a G^a$

$V(\mathbf{\Phi} \cdot \mathbf{\Phi})$ is a potential such that the vacuum expectation of Φ is non-zero. When a specific form of $V(\mathbf{\Phi} \cdot \mathbf{\Phi})$ is required we use

$$V(\mathbf{\Phi} \cdot \mathbf{\Phi}) = \frac{\lambda}{8}(\mathbf{\Phi} \cdot \mathbf{\Phi} - v^2)^2 \, .$$

How to define $F_{\mu\nu}$

Usual Definition $\qquad F_{\mu\nu} \equiv \boldsymbol{F}_{\mu\nu} \cdot \boldsymbol{H}$

$\boldsymbol{H} \in$ Cartan subalgebra

$dF \neq 0$ $\qquad F_{\mu\nu} = \boldsymbol{F}_{\mu\nu} \cdot \bar{\boldsymbol{\Phi}}$

Bianchi identity not satisfied

t'Hooft – $SO(3)$

$$F_{\mu\nu} = \boldsymbol{F}_{\mu\nu} \cdot \bar{\boldsymbol{\Phi}} - \frac{1}{ie} D_\mu \bar{\boldsymbol{\Phi}} \wedge D_\nu \bar{\boldsymbol{\Phi}} \cdot \bar{\boldsymbol{\Phi}}$$

$dF = 0$ iff $\qquad \bar{\boldsymbol{\Phi}} \wedge E_\alpha = \pm E_\alpha$ or 0

(Bianchi identity satisfied)

Here E_α is any one of the generators of the Lie algebra.

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Make the ansatz that the Higgs field $\mathbf{\Phi}$ and vector potential $\boldsymbol{A}$ take the form

$$\mathbf{\Phi} = \left(Q(r)\,\alpha_1 T_z + \alpha_2 T_\perp\right) v\,,$$
$$\boldsymbol{A} = \frac{g_e}{g}\, S(r)\, v\, \alpha_1\, T_z\, dt \;+\; T_z\, (-C)\, W(r)(1-\cos\theta)\, d\phi\,,$$

where

$$\begin{aligned} &W(r), Q(r), S(r) \;\to 0\,, && \text{as} \quad r \to 0\,; \\ &W(r), Q(r) \;\to 1\,,\; S(r) \;\to\; 1 - \frac{g}{g_m\, e\, \alpha_1\, v\, r}\,, && \text{as} \quad r \to \infty\,; \\ &g = \sqrt{g_e^2 + g_m^2}\,. \end{aligned}$$

Here C is an arbitrary constant, and quantities g_e and g_m are the electric and magnetic charges.

▸ root system – $SU(3)$

Applying the gauge transformation

$$\chi = e^{-i\phi T_z} e^{-i\theta T_y} e^{i\phi T_z}$$

to $\boldsymbol{A}$ and $\boldsymbol{\Phi}$ we obtain

$$\begin{aligned} \boldsymbol{A} &\rightarrow \chi \boldsymbol{A} \chi^{-1} - \frac{1}{ie} d\chi \, \chi^{-1} \\ &= \frac{g_e}{g} S(r) \, v \, \alpha_1 \, T_r \, dt + \frac{W(r)}{e} \left(T_\theta \, \sin\theta \, d\phi - T_\phi \, d\theta \right) . \end{aligned}$$

and

$$\begin{aligned} \boldsymbol{\Phi} &\rightarrow \chi \boldsymbol{\Phi} \chi^{-1} \\ &= v \left[\alpha_2 \, T_\perp + Q(r) \, \alpha_1 \, T_r \right] . \end{aligned}$$

We have used the fact that

$$d\chi \, \chi^{-1} = -i \left[(1 - \cos\theta) \, T_r \, d\phi + \sin\theta \, T_\theta \, d\phi - T_\phi \, d\theta \right] .$$

$$
\begin{aligned}
\text{magnetic charge} \qquad g_m &= \frac{4\pi}{|\alpha|^2 e} \\
\text{in general} \qquad &= n_m \frac{4\pi}{|\alpha|^2 e} \qquad (n_m \in Z) \\
\text{electric charge} \qquad g_e &= n_e h_{N-1} e = n_e \frac{\alpha_1}{N} e \\
&= n_e \frac{1}{N} \sqrt{\frac{N}{2(N-1)}}\, e \\
&= n_e\, e' \,,
\end{aligned}
$$

where n is an integer. The electric charge quantization is derived from the fundamental representation where $h_{N-1} = \frac{\alpha_1}{N}$.

Magnetic/electric charge (fundamental unit) relationship

$$SU(N), (|\alpha|^2 = 1) \qquad g_m\, g_e = n_e\, 4\pi\, \frac{1}{N} \sqrt{\frac{N}{2(N-1)}}$$

$$G2_E, (|\alpha|^2 = 1/3) \qquad g_m\, g_e = n_e\, \frac{4\pi}{2}\, 3 \qquad (e' = \frac{e}{2})$$

$$\text{t'Hooft/Polykov} \qquad g_m\, g_e = n_e\, 4\pi \qquad (e' = e)$$

$$\text{Dirac} \qquad g_m\, g_e = n_e\, \frac{4\pi}{2} \qquad (e' = e)$$

Magnetic Monopole Mass ($V = 0$)

$$SU(N), (|\alpha|^2 = 1) \qquad M = \frac{g}{e} \sqrt{\frac{N}{2(N-1)}}\, M_X$$

$$G2_E, (|\alpha|^2 = 1/3) \qquad M = \frac{g}{2e}\, M_X$$

$$\text{t'Hooft/Polykov} \qquad M = \frac{g}{e}\, M_X$$

Dirac —

▸ root system – $G2$

Montenon - Olive Conjecture

	Mass	$(g_e,\ g_m)$	Spin
Higgs	0	(0, 0)	0
Photon	0	(0, 0)	1
$W^{\pm}$	v e	(e, 0)	1
M	vg	(0, g)	0

Table: The gauge group $SO(3)$

Langrangian Density ($V = 0$)

Witten Effect: $$\mathcal{L}_\theta = -\frac{\theta e^2}{32\pi^2} * \mathbf{F}_{\mu\nu} \cdot \mathbf{F}^{\mu\nu} = \frac{\theta e^2}{8\pi^2} \mathbf{E}^i \cdot \mathbf{B}^i$$

Traditional: $$\mathcal{L}_{\text{trad}} = -\frac{1}{4} \mathbf{F}_{\mu\nu} \cdot \mathbf{F}^{\mu\nu} = \frac{1}{2} (\mathbf{E}^i \cdot \mathbf{E}^i - \mathbf{B}^i \cdot \mathbf{B}^i)$$

Consider U(1) gauge transformations about $\bar{\mathbf{\Phi}}$:

$$\delta \boldsymbol{A}_\mu = \mathcal{D}_\mu \bar{\mathbf{\Phi}}$$

Let $\boldsymbol{\eta}$ be the generator of infinitesimal gauge transformations. Since

physical quantities require $e^{i2\pi\boldsymbol{\eta}} = 1$

eigenvalues of $\boldsymbol{\eta}$ $\eta = n, (n \in Z)$

By Noether's theorem $$\eta = \frac{\partial \mathcal{L}}{\partial \partial_0 A_\mu^\alpha} \delta A_\mu^\alpha$$

Therefore $$\eta = \frac{g_e}{e} + \frac{\theta e g_m}{8\pi^2}$$

Consequently ($n_m = e g_m / 4\pi$) $$g_e = n\, e - \frac{\theta}{2\pi}\, n_m\, e$$

$SL(2,Z)$

Define

$$\tau = \frac{\theta}{2\pi} + i\frac{4\pi}{e^2}$$

$$\theta \to \theta + 2\pi \qquad \tau \to \tau + 1$$

$$(e \to g_m \equiv \frac{4\pi}{e} \qquad \tau \to -\frac{1}{\tau}$$

$$\theta = 0)$$

$$\tau \to \frac{a\tau + b}{c\tau + d} \qquad a, b, c, d \in Z, ad - bc = 1$$

$$\begin{pmatrix} n \\ n_m \end{pmatrix} \to \begin{pmatrix} a & -b \\ c & -d \end{pmatrix} \begin{pmatrix} n \\ n_m \end{pmatrix}$$

Magnetic Charge	$g_m = \frac{4\pi}{e} n_m$
Electric Charge	$g_e = ne - n_m \frac{\theta}{2\pi} e$
Mass of Dyon	$M^2 \geq v^2(g_e^2 + g_m^2)$

$$\begin{aligned}\mathcal{L} =& \frac{1}{4}\mathbf{F}^{\mu\nu}\cdot\mathbf{F}_{\mu\nu} - \frac{\theta e^2}{32\pi^2}\mathbf{F}^{\mu\nu}\cdot *\mathbf{F}_{\mu\nu} - \frac{1}{2}\mathcal{D}^{\mu}\mathbf{\Phi}\cdot\mathcal{D}_{\mu}\mathbf{\Phi} \\ \equiv& -\frac{1}{32\pi}\mathrm{Im}(\tau)(\mathbf{F}^{\mu\nu} + i * \mathbf{F}^{\mu\nu})\cdot(\mathbf{F}_{\mu\nu} + i * \mathbf{F}_{\mu\nu}) \\ & -\frac{1}{2}\mathcal{D}^{\mu}\mathbf{\Phi}\cdot\mathcal{D}_{\mu}\mathbf{\Phi}\end{aligned}$$

Summary

- Quantum corrections would be expected to generate a non-zero potential $V(\Phi)$ even if one is absent classically and should also modify the classical mass formula. Thus there is no reason to think that the duality of the spectrum should be maintained by quantum corrections.
- The W bosons have spin one while the monopoles are rotationally invariant indicating that they have spin zero. Thus even if the mass spectrum is invariant under duality, there will not be an exact number of matching states and quantum numbers.
- The proposed duality symmetry seems impossible to test since rather than acting as a symmetry of a single theory it relates two different theories, one of which is necessarily at strong coupling where we have little control of the theory.

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Summary (continued)

- The $N = 4$ Super Yang-Mills theory solves the first problem because of exact quantum scale invariance ($V = 0$). It also solves the second problem because additional particles are added to the spectrum.

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Do we really need the Higgs field?

Assume an extra dimension Let $A_5 = \Phi$. Assume $V = 0$.

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} \cdot F^{\mu\nu} + \frac{1}{2} D_\mu A_5 \cdot D^\mu A_5 \,,$$
$$= -\frac{1}{4} F_{MN} \cdot F^{MN} \; (M, N = 0 \ldots 5)$$

where

$$F_{\mu 5} = \partial_\mu A_5 - ie\, A_\mu \wedge A_5$$

The Higgs field becomes a component of the Yang-Mills potential in the extra dimension.

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to be continued ...

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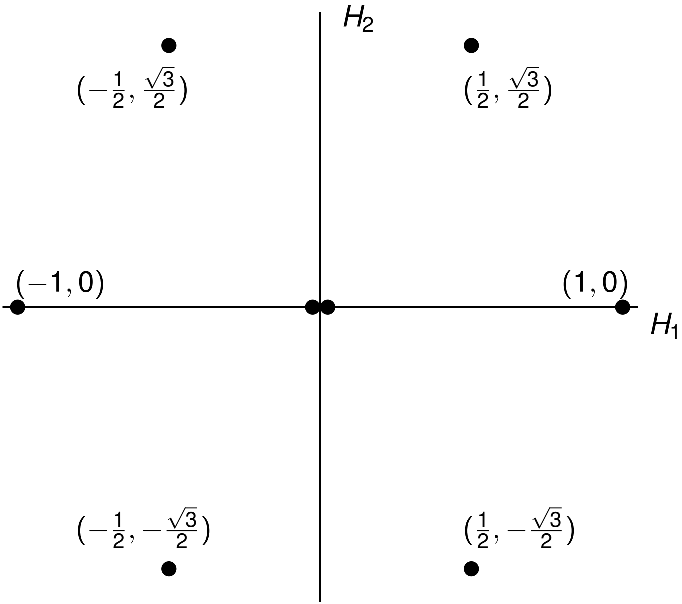
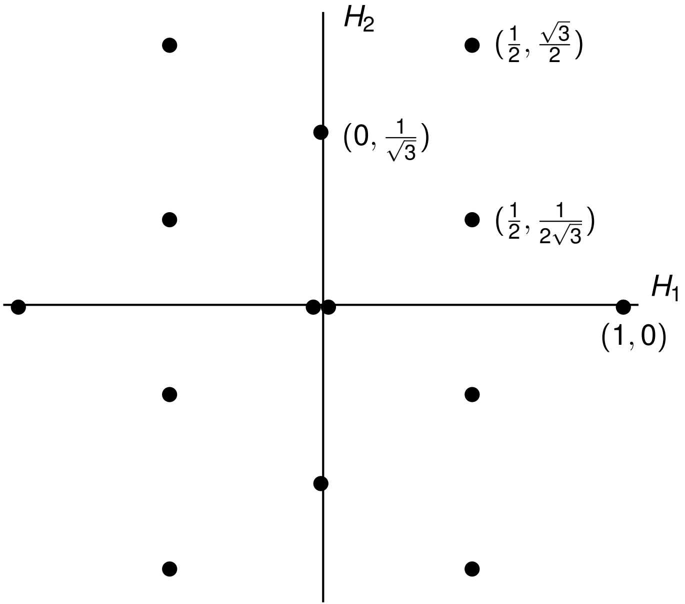
monopole

SU(3) Root System



monopole

G2 Root System



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