Snell's Law

Introduction

According to Snell's Law [1] when light is incident on an interface separating two media, as depicted in Figure 1, the angles of incidence and refraction, θ_1 and θ_2, are related,

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2) \; , \tag{1}$$

where θ_1 or θ_2 is the angle between the normal to the interface and the direction in which the incident or refracted wave is propagating. The quantities n_1 and n_2 are the indices of refraction of the two media.

The purpose of this experiment is to determine experimentally the index of refraction, relative to air, of a cubical block of plastic. The experiment involves measuring angles of incidence and refraction and then using Eq. 1 to calculate the index of refraction.

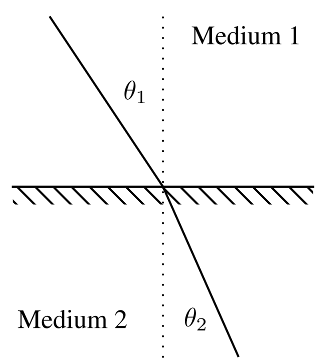

Figure 1: A geometrical depiction of Snell's Law.

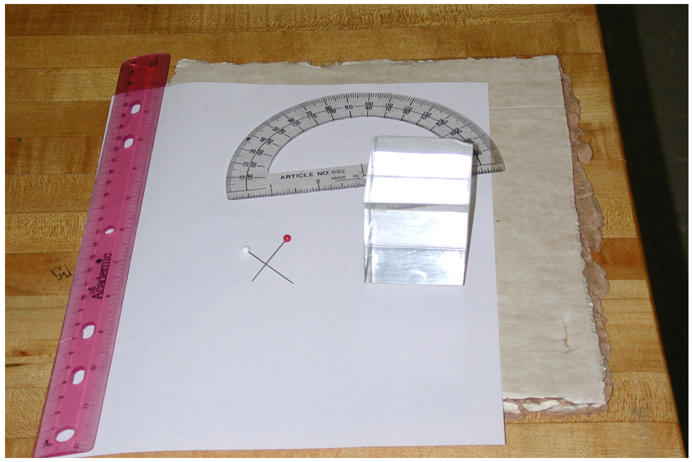

Figure 2: Experimental arrangement.

Procedure

Place the worksheet (Figure 4) on the piece of cardboard. Insert five common pins into the circles denoted 0 through 4 on the worksheet.[1] Now place the plastic cube on the square outlined on the worksheet. The setup should appear as in Figure 2. Look through the face of the cube where pin 1 is positioned. You should be able to view pin 0, which is located on the opposite face of the cube. Now, sighting along the line connecting pin 1 to pin 0, place a third pin directly behind pin 0 so that it is directly hidden from view. Move this pin approximately three inches away from pin 0 in such a manner that it remains hidden by pin 0. Insert this pin into the cardboard, and label it pin $1'$. The three pins should appear to lie on a straight line (See Fig 3.). Apply the same procedure using pin 2 and pin 3, labelling the newly inserted pins as $2'$ and $3'$. Remove the cube from the worksheet. On the worksheet draw three straight lines, one connecting pin 0 to pin $1'$, one connecting pin 0 to pin $2'$, and one connecting pin 0 to pin $3'$. Each pair of lines pin i ($i = 1, 2, 3$) – pin 0 and pin 0 – pin i' should resemble the pair of lines corresponding to the incident and refracted waves in Figure 1.

1. For each pair of connecting lines obtain the angle of incidence θ_1 and angle of refraction θ_2.

2. For each pair of angles, calculate the relative index of refraction n of the plastic: $n = \frac{n_2}{n_1} = \frac{\sin(\theta_1)}{\sin(\theta_2)}$.

3. Compute the average and standard error of the three values of the relative index of refraction, as discussed in the Appendix. In Table 1 Report the index of refraction in accord with Eq. 5.

[1]The purpose of pin 4 is to prevent the cube from moving while data are being collected.

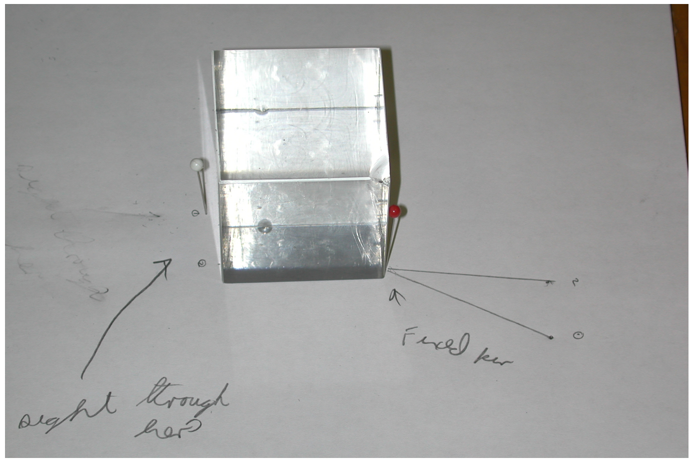

Figure 3: The experiment in progress.

4. In Table 2 report the 95% confidence interval for the index of refraction, in accord with Eq's 6, 7, and 8.

Appendix

Given a set of data x_i ($i = 1 \ldots N$) corresponding to a quantity whose true value is x_t. If each of the x_i differs from x_t because each x_i includes a random error ϵ_i, i.e. $x_i = x_t + \epsilon_i$, then an unbiased estimate of x_t is $\bar{x}$,

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i \; , \tag{2}$$

and an unbiased estimate of its standard error is σ,

$$\sigma = \frac{\sigma_{N-1}}{\sqrt{N}} \; , \tag{3}$$

where

$$\sigma_{N-1} = \sqrt{\frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{N-1}} \; . \tag{4}$$

In calculating σ_{N-1}, the number of degress of freedom $\nu = N - 1$ is used rather than N. Note: In Microsoft Excel $\bar{x}$ and σ_{N-1} can be calculated using the library functions AVERAGE and STDEV.

To reflect the statistical uncertainty in x_t, the experimental results are typically reported as

$$Q_{\text{est}} \pm \delta Q \; , \tag{5}$$

where $Q_{\text{est}} = \bar{x}$ is the unbiased estimate of x_t and $\delta Q = \sigma$ is the standard deviation of $\bar{x}$. Equation 5 can be understood informally to mean that, assuming the experimental results are consistent with theory, then the value of Q_{est}, predicted by theory, is likely to lie within the limits defined by Equation 5. This informal interpretation can be made more precise. Specifically, one specifies a confidence interval, e.g. the 95% confidence interval (See below.). Then assuming that the theory accounts for the experimental results, there is a 95% probability that the calculated confidence interval from some future experiment encompasses the theoretical value. If, for a given experiment the value of x_t predicted by theory lies outside of the confidence interval, the assumption that theory accounts for the results of the experiment is rejected, i.e. the experimental results are inconsistent with theory. The confidence interval is expressed as

$$[Q_{\text{est}} - X\ \delta Q, Y_{\text{est}} + X\ \delta Q]\ , \tag{6}$$

The quantity X is obtained from a Student's t-distribution and depends on the confidence interval and the degrees of freedom. A detailed and illuminating discussion of the Student's t-distribution can be found in the Wikipedia on-line free encyclopedia. [2] There are various ways of obtaining or calculating the value of X. For example, the spreadsheet Microsoft Excel includes a library function T.INV.2T for calculating X based on a two-tailed t-test. Specifically,

$$X = \text{T.INV.2T}(p, \nu)\ , \tag{7}$$

where the probability $p = 1 - \frac{\text{(the confidence interval)}}{100}$ and ν is the degrees of freedom. Consider the following example for illustrative purposes. The number of data points is 4; the confidence interval is 95%. Therefore

$$X = \text{T.INV.2T}(1 - \frac{95}{100}, 3 - 1) = 4.303\ . \tag{8}$$

References

[1] Wikipedia. Snell's law. http://en.wikipedia.org/wiki/Snell's_law, 2008. [Online; accessed 12-March-2008].

[2] Wikipedia. Student's t-distribution. https://en.wikipedia.org/wiki/Student%27s_t-distributi 2017. [Online; accessed 22-March-2017].

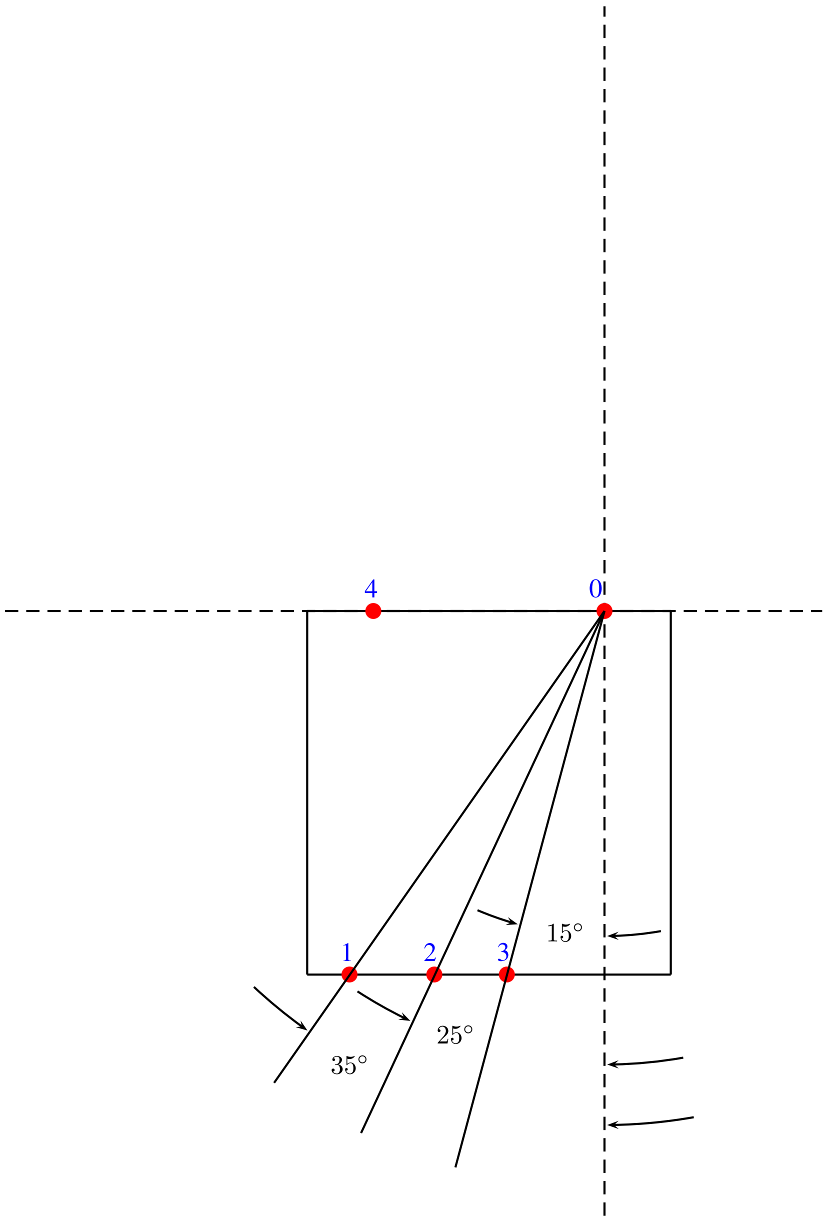

Figure 4: Worksheet.

θ_1	θ_2	n
	$15°$	
	$25°$	
	$35°$	
$\bar{n} \pm \sigma$		

Table 1: Data and Calculations

$\bar{n} - X\sigma$	$\bar{n} + X\sigma$

Table 2: The 95% Confidence Interval