Resistors in Combination

Introduction

The concept of equivalent resistance can be useful in the analysis of some electric circuits. The equivalent resistance of a combination of resistors connected between two points (A and B) is the resistance of that single resistor which produces the same effect as the combination of resistors. Specifically, for a given potential difference between A and B, the current at A or B is the same for the equivalent resistor as for the combination of resistors. Two notable examples of such are shown in Figures 2 and 3. The equivalent resistance, R_s of resistors connected in series is given by

$$R_\mathrm{s} = R_1 + R_2 \; . \tag{1}$$

For resistors in parallel the equivalent resistance, R_p satisfies the relationship

$$\frac{1}{R_\mathrm{p}} = \frac{1}{R_2} + \frac{1}{R_3} \; , \tag{2}$$

so that

$$R_\mathrm{p} = \frac{R_2 R_3}{R_2 + R_3} \; . \tag{3}$$

A primary motivation for studying ciruits containing either two resistors in series or parallel from the myriad of possibilities is that many ciruits can be reduced to combinations of these two elemental configurations. Consider the more complicated combination of resistors in Figure 4. This combination is equivalent to a single resistor in series with a parallel combination of two other resistors. The equivalent resistance, R_c is given by

$$R_\mathrm{c} = R_1 + \frac{R_2 R_3}{R_2 + R_3} \; . \tag{4}$$

An example of a circuit which cannot be reduced to series or parallel combinations of resistors is the bridge network shown in Figure 5. It can be shown that the equivalent resistance of this circuit, R_b, is given by

$$R_\mathrm{b} = \frac{T}{B} \; , \tag{5}$$

where

$$\begin{aligned} T =& R_2R_5R_1 + R_2R_3R_1 + R_2R_3R_5 + R_4R_5R_2 + \\ &+ R_4R_5R_1 + R_4R_3R_1 + R_4R_3R_5 + R_2R_4R_1 \; , \end{aligned} \tag{6}$$

and

$$B = R_2 R_3 + R_2 R_4 + R_5 R_2 + R_3 R_1 + \\ + R_3 R_5 + R_5 R_1 + R_4 R_3 + R_4 R_1 \ . \tag{7}$$

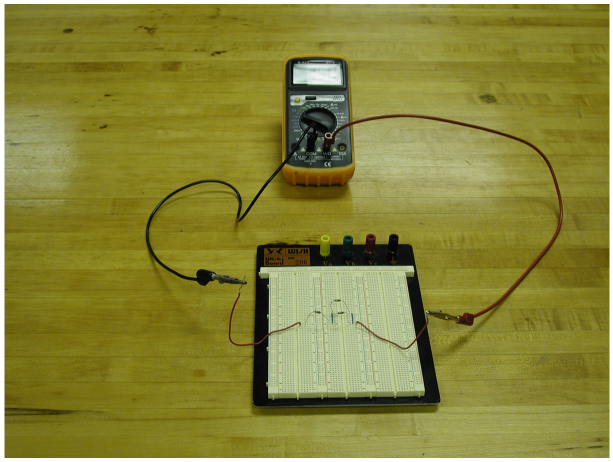

Figure 1: Shown is the equipment used in performing the experiment. It includes a multimeter, a power supply, a breadboard, and resistors in combination. The particular combination, comprising one resistor in series with a parallel combination of two, is one of the arrangements of resistors being studied in the experiment.

Procedure

Record all measurements and calculations in Table 1.

1. Using the multimeter as an ohm meter measure the resistance of each of the five resistors. The values of resistance are: $R_1 \approx 390\,\Omega$, $R_2 \approx 620\,\Omega$, $R_3 \approx 100\,\Omega$, $R_4 \approx 470\,\Omega$, and $R_5 \approx 750\,\Omega$.
2. Construct the series circuit, as in Figure 2. Measure its equivalent resistance.
3. Construct the parallel circuit, as in Figure 3. Measure its equivalent resistance.
4. Construct the combination circuit, as in Figure 4. Measure its equivalent resistance.
5. Construct the bridge network, as in Figure 5. Measure its equivalent resistance.
6. For each case calculate the theoretical values of the equivalent resistances using Equations 1, 3, 4, or 5, as appropriate.
7. Do the theoretical values appear to be consistent with the measured values?

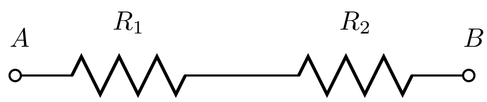

Figure 2: Resistors in Series

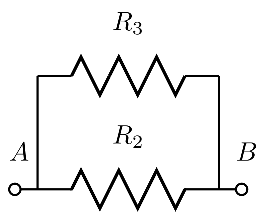

Figure 3: Resistors in Parallel

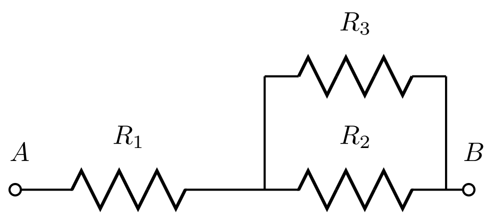

Figure 4: Series and Parallel Combination of Resistors

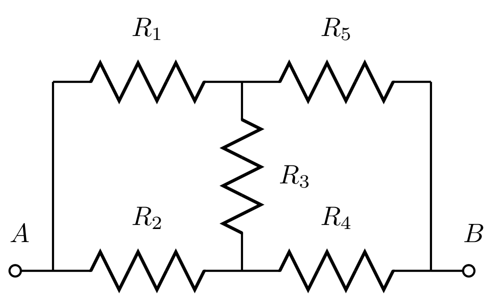

Figure 5: Bridge Network

Experiment							Theory
	R_1	R_2	R_3	R_4	R_5	R_{equiv}	R_{equiv}
Series							
Parallel							
Combination							
Bridge Network							

Table 1: Data and Calculations