The Focal Length of a Thin Converging Lens

Introduction

The ability of a lens to focus light is a consequence of its shape and optical density relative to that of the surrounding environment. For example, a thin converging lens, typically made from some type of glass, is fabricated into the shape of two spherical caps of relatively small curvature with the convex side of each cap facing outwardly (See point O in Fig. 1.). The focal point of such a lens can be defined as the point where light from a distant source is focused by the lens, and its focal length as the distance from the lens to the focal point (An informative and illuminating discussion of lenses can be found in the Wikipedia [1].). The purpose of this experiment is to determine the focal length of a thin converging lens.

Because of its ability to focus light a converging lens can produce an image of an object, as represented in Fig. 1. The object and image are denoted by the line segments $\overline{AB}$ and $\overline{A'B'}$. The red lines are a representative sample of light rays emanating from point B. The lens redirects and focuses the light rays at point B'. In this example the image is characterized as real and inverted. It is real because a viewer to the right of the lens would observe light rays originating from the image, as is shown for the image point B'. The focal points on either side of the lens are denoted F and F'. Whenever the two spherical caps have the same curvature, as in Fig. 1, the distance from the lens to either focal point is the same. Thus, there is a single focal length denoted f which equals the distance $\overline{OF} = \overline{O'F'}$. One can show that the focal length is related to the following quantites,

$$f = \frac{dd'}{d + d'} \, , \tag{1}$$

where the object distance d equals $\overline{OA}$, and the image distance d' equals $\overline{O'A'}$. In Eq. 1 taking the limit as d approaches infinity, which corresponds in practice to locating the object far from the lens, one obtains

$$f = \lim_{d \to \infty} \frac{dd'}{d + d'} = d' \, , \tag{2}$$

showing that light coming from a distant source is focused at the focal point of the lens. Also, one can show

$$f = \frac{d'}{1 - m} \, , \tag{3}$$

where the magnification m is defined as

$$m \equiv \frac{h'}{h} \, . \tag{4}$$

Here the object height h equals the distance $\overline{AB}$, and the image height h' equals minus the distance $\overline{A'B'}$. The reason $h' < 0$ is that the image is inverted relative to the object.
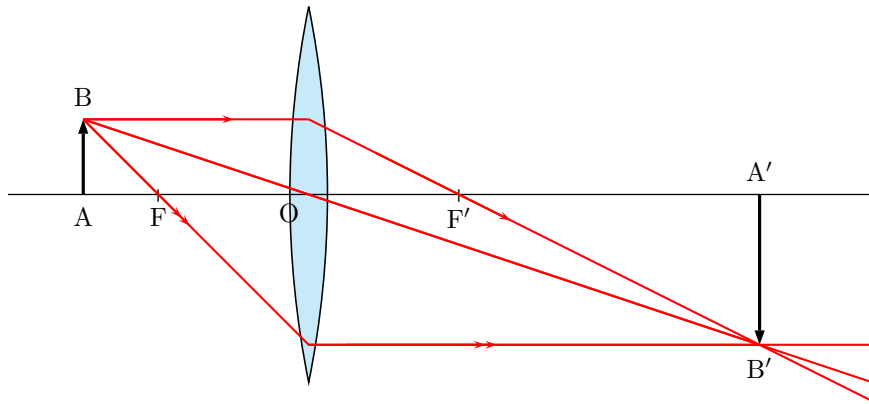

Figure 1: A thin converging lens producing a real inverted image.

Figure 2: The optical bench. From left to right is the object, which consists of a light source and template, the converging lens, and a screen on which the image is projected.

Procedure

Attach the light source (the object) to the optical bench at the 5 cm mark. Attach the lens to the optical bench at the 35 cm mark. Attach the screen to the optical bench. The configuration should resemble that of Fig. 2. Connect the light source to the ac outlet.

1. Case 1: Move the screen to the location where a well defined image is formed. Record the object distance, the image distance, the object height, and the image height in Table 1 (Note: the image height is a negative number because the image is inverted.).

2. Case 2: Move the lens to the 50 cm mark. Move the screen to the location where a well defined image is formed. Record the object distance and the image distance in Table 1. The object and image heights are not required.

3. Case 3: Move the lens to the 65 cm mark. Adjust the screen until a well defined image is formed. Record the object distance and image distance in in Table 1.

4. Case 4: Unplug the light source and remove it from the optical bench. Obtain an image of a distant object. This may require rotating the optical bench to *aim* the lens at the object. Move the screen to the location where a well defined image is formed. Record the image distance in Table 1.

Method 1

Using Eq. 1 or 2, as appropriate, calculate the focal length of the lens for each of the four cases. Record the values in Table 2. Calculate the average and standard error of these values and report the value of the focal length in Table 2 in the form given by Eq. 8.

Method 2

For Case 1, calculate the magnification of the lens using Eq. 4. Substitute the value of the magnification and the image distance into Eq. 3 to obtain another estimate of the focal length of the lens. Record its value in Table 2. According to the criterion Eq. 9 are the focal lengths obtained by the two methods consistent?

Appendix

Given a set of data x_i $(i = 1 \ldots N)$ corresponding to a quantity whose true value is x_t. If each of the x_i differs from x_t because each x_i includes a random error ϵ_i, i.e. $x_i = x_t + \epsilon_i$, then an unbiased estimate of x_t is $\bar{x}$,

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i \,, \tag{5}$$

3

and an unbiased estimate of its standard error is σ,

$$\sigma = \frac{\sigma_{N-1}}{\sqrt{N}} \ , \tag{6}$$

where

$$\sigma_{N-1} = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N-1}} \ . \tag{7}$$

In calculating σ_{N-1}, the number of degress of freedom $\nu = N - 1$ is used rather than N. Note: In Microsoft Excel $\bar{x}$ and σ_{N-1} can be calculated using the library functions AVERAGE and STDEV.

To reflect the statistical uncertainty in x_t, the experimental results are typically reported as

$$Q_{\text{est}} \pm \delta Q \ , \tag{8}$$

where $Q_{\text{est}} = \bar{x}$ is the unbiased estimate of x_t and $\delta Q = \sigma$ is the standard deviation of $\bar{x}$. Equation 8 can be understood informally to mean that, assuming the experimental results are consistent with theory, then the value of Q_{est}, predicted by theory, is likely to lie within the limits defined by Equation 8. This informal interpretation can be made more precise. Specifically, one specifies a confidence interval, e.g. the 95% confidence interval (See below.). Then assuming that the theory accounts for the experimental results, there is a 95% probability that the calculated confidence interval from some future experiment encompasses the theoretical value. If, for a given experiment the value of x_t predicted by theory lies outside of the confidence interval, the assumption that theory accounts for the results of the experiment is rejected, i.e. the experimental results are inconsistent with theory. The confidence interval is expressed as

$$[Q_{\text{est}} - X \ \delta Q, Y_{\text{est}} + Q \ \delta Q] \ , \tag{9}$$

The quantity X is obtained from a Student's t-distribution and depends on the confidence interval and the degrees of freedom. A detailed and illuminating discussion of the Student's t-distribution can be found in the Wikipedia on-line free encyclopedia. [2] There are various ways of obtaining or calculating the value of X. For example, the spreadsheet Microsoft Excel includes a library function T.INV.2T for calculating X based on a two-tailed t-test. Specifically,

$$X = \text{T.INV.2T}(p, \nu) \ , \tag{10}$$

where the probability $p = 1 - \frac{\text{(the confidence interval)}}{100}$ and ν is the degrees of freedom. Consider the following example for illustrative purposes. The number of data points is 4; the confidence interval is 95%. Therefore

$$X = \text{T.INV.2T}(1 - \frac{95}{100}, 4 - 1) = 3.182 \ . \tag{11}$$

References

[1] Wikipedia. Lens (optics). `http://en.wikipedia.org/wiki/Lens_(optics)`, 2008. [Online; accessed 14-March-2008].

[2] Wikipedia. Student's t-distribution. `https://en.wikipedia.org/wiki/Student%27s_t-distribution`, 2017. [Online; accessed 22-March-2017].

Case	d (cm)	d' (cm)	h (cm)	h' (cm)
1				
2				
3				
4	∞			

Table 1: Data

Case	Method 1	Method 2
1		
2		
3		
4		
f (cm)	$\pm$	

Table 2: Calculations