# Determination of the Earth's Magnetic Field

## Introduction

Although historically ancient travelers made abundant use of the earth's magnetic field for the exploration of the earth, they were ignorant of its origin. In many respects the earth's magnetic field exhibits characteristics similar to those of a bar magnet; nonetheless, the mechanisms responsible for generating each are vastly different. A detailed and illumintating discussion of the earth's magnetic field, including its origin, can be found in the Wikipedia online encyclopedia. Magnetic field lines appear to originate near the south geographic pole, i.e. magnetic north pole, and terminate near the north geographic pole, i.e. magnetic south pole. It is interesting to note that in the vicinity of Wilmington, North Carolina the magnetic field lines enter the earth at a relatively steep angle. The angle of inclination or dip angle, which is the angle that a compass needle makes with respect to the plane of the horizon, is approximately $60°$. In this experiment principles of magnetostatics and elementary vector analysis are used to determine the earth's magnetic field in the vicinity of Wilmington, North Carolina. The primary equipment used in performing the experiment is shown in Fig. 1.

## Methodology and Procedure

As depicted in Figure 2 the earth's magnetic field $B_e$ can be decomposed into a component $B_h$ which is parallel to the plane of the horizon and a component $B_v$ which is perpendicular to the plane of the horizon. Thus, $B_e$ and $B_h$ are related by

$$B_e = \frac{B_h}{\cos(\theta_i)} , \tag{1}$$

where $\theta_i$ is the angle of inclination. If a compass needle is subjected to a known external magnetic field $B_x$ which acts perpendicularly to $B_h$, the compass needle will deflect through an angle $\theta_x$ away from magnetic south (See Figure 3.). Consequently, $B_h$ is related to $B_x$ by

$$B_h = \frac{B_x}{\tan(\theta_x)} . \tag{2}$$

Thus, Eq's 2 and 1 relate the earth's magnetic field, which is unknown, with the magnetic field $B_x$, which is known in principle.

The tangent galvanometer is the primary piece of equipment used in performing the experiment. It consists of a compass positioned at the center of a coil of wire. The coil

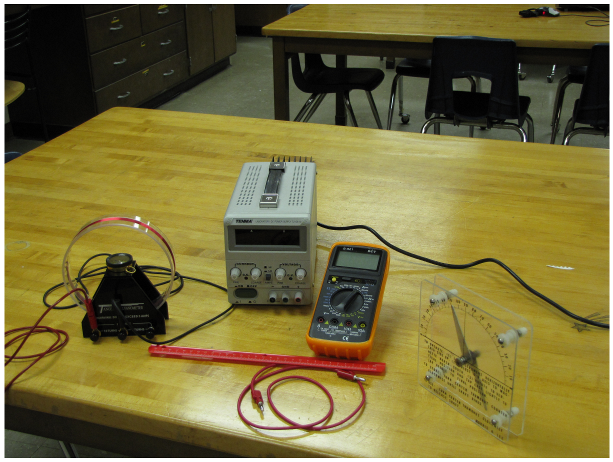

Figure 1: The equipment used in performing the experiment includes (in the foreground) a banana plug and (in the background from left to right) a tangent galvanomenter with attached banana plugs, a ruler, a power supply, an ammeter, and a compass for measuring the angle of inclination of the earth's magnetic field.

is connected to a variable power supply with an ammeter inserted into the circuit for monitoring the current through the coil. Whenever there is a current through the coil, a magnetic field is produced perpendicular to the plane of the coil. The magnitude of the magnetic field $B_x$, in units of microtesla, at the center of the coil is

$$B_x = N\frac{\mu_0 I}{D} \times 10^6 = N\frac{(4\pi \times 10^{-1})I}{D}\,, \tag{3}$$

where $\mu_0 = 4\pi \times 10^{-7}$, $N$ is the number of turns which the coil comprises, $D$ is the diameter of the coil measured in meters, and $I$ is the current through the coil measured in amps.

The procedure for performing the experiment is, as follows.

1. Measure the angle of inclination, $\theta_i$, using the compass (See Fig. 1.). Record its value in Table 3.

2. Measure the diameter of the coil, $D$, and record its value in Table 1.

3. Assemble the circuit consisting of the power supply, the ammeter set on the 10 amp scale, and the tangent galvanometer. Connect the circuit to the two terminals of the galvanometer which correspond to ten turns ($N = 10$), i.e. the middle terminal and the terminal to its left.

4. It may be prudent to apply preliminary adjustments to the power supply to protect the ammeter or tangent galvanometer from damage. See Appendix 1 for details.

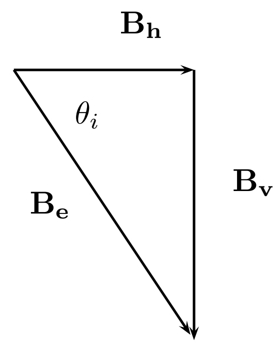


Figure 2: The horizontal and vertical components of the earth's magnetic field. The angle $\theta_i$ is the angle of inclination.

5. Align the plane of the coil so that the direction of the magnetic field produced by the coil is perpendicular to that of the earth's magnetic field. There are a number of ways of accomplishing this, a relatively straightforward and unsophisticated example of which now follows. First, align the plane of the coil so that its normal points, approximatiely, in the direction of the north pole of the compass needle. Set the fine/coarse voltage settings of the power supply to zero. Turn on the power supply. Slowly increase the fine voltage setting on the power supply. If the compass needle deflects, reduce the voltage setting to zero, and realign the plane of the coil so that when the fine voltage setting is again increased, the compass needle does not deflect. Reset the the fine voltage setting to zero. Using the angular markings on the compass, rotate the coil $90°$ with respect to the compass needle. Thus, any magnetic field generated by the coil will be perpendicular to to the horizontal component of the earth's magnetic field.

6. With the fine/coarse voltage settings at zero, slowly increase the fine or coarse voltage settings until the compass needle deflects $45°$, i.e. $\theta_x = 45°$ . Reverse the polarity of the connections at the power supply. The compass needle should deflect approximately $45°$, in the opposite direction. If this is not the case, re-adjust the alignment of the coil, and redo this procedure until the compass needle deflects $45°$ in both directions. Report the current $I$ in the approprate row in Table 2. Using Equations 3 and 2 calculate $B_x$ and $B_h$. Record their values in Table 2.

7. Perform the previous step for angles of $30°$, $60°$, and $75°$. Omit that part of the procedure in which the polarity of the connections at the power supply are reversed.

8. Report the average of the $B_h$ values and its standard error in Table 3 (See Eq. 7

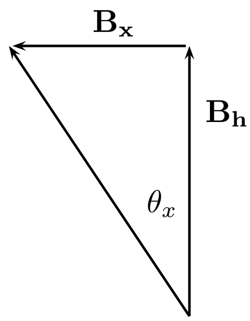


Figure 3: The horizontal component of the earth's magnetic field and the perturbing external field of the tangent galvanometer.

in the Appendix.).

9. Using Eq. 1 calculate the value of the earth's magnetic field $B_e$ using the best estimate of $B_h$, i.e. the average of the $B_h$ values. Report $B_e$ in Table 3.

10. Compare the experimental values of $\theta_i$, $B_h$, and $B_e$, to those of the National Geophysical Data Center (NGDC). Access the NGDC website. Enter the zip code of Wilmington North Carolina, i.e. 28403, into the search field. Click on "Get Location." Scroll down to "Magnetic component." Highlight "I(nclination", and click on "Compute Magnetic Field Components." Report the value of $\theta_i$ in Table 3. Click on the back arrow of the web browser to return to the previous webpage. Highlight "H(orizontal Intensity)", and click on "Compute Magnetic Field Components." Report the value of $B_h$ in Table 3. Again, return to the previous webpage. Highlight "F(Total Intensity)", and click on "Compute Magnetic Field Components." Report the value of $B_e$ in Table 3. Note: the NGDC values of $B_h$ and $B_e$ are given in units of nT and should be converted to $\mu$T.

11. According to the criterion given in Appendix 2, i.e. Eq. 8, is the experimental value of $B_h$ in agreement with the NGDC value?

# 1 Appendix

The following preliminary adjustment to the power supply may be useful in preventing damage to the ammeter or tangent galvanometer when performing the experiment.

1. Be certain that the amp button is set to low. It should be pushed in.

2. Set the coarse and fine voltage settings to zero.

3. Turn on the power supply.
4. Set the coarse voltage setting to full scale.
5. Slowly increase the fine or coarse current setting until the ammeter reads approximately 1.5 amps.
6. Adjust the coarse voltage setting to zero.
7. When performing the experiment adjustments to the current should be made using only the fine/coarse voltage setting on the power supply. No further adjustments should be made to the fine/coarse current setting. Proceed with the experiment.

# 2 Appendix

Given a set of data $x_i$ ($i = 1 \ldots N$) corresponding to a quantity whose true value is $x_t$. If each of the $x_i$ differs from $x_t$ because each $x_i$ includes a random error $\epsilon_i$, i.e. $x_i = x_t + \epsilon_i$, then an unbiased estimate of $x_t$ is $\bar{x}$,

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i \ , \tag{4}$$

and an unbiased estimate of its standard error is $\sigma$,

$$\sigma = \frac{\sigma_{N-1}}{\sqrt{N}} \ , \tag{5}$$

where

$$\sigma_{N-1} = \sqrt{\frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{N-1}} \ . \tag{6}$$

In calculating $\sigma_{N-1}$, the number of degress of freedom $\nu = N - 1$ is used rather than $N$. Note: In Microsoft Excel $\bar{x}$ and $\sigma_{N-1}$ can be calculated using the library functions AVERAGE and STDEV.

To reflect the statistical uncertainty in $x_t$, the experimental results are typically reported as

$$Q_{\text{est}} \pm \delta Q \ , \tag{7}$$

where $Q_{\text{est}} = \bar{x}$ is the unbiased estimate of $x_t$ and $\delta Q = \sigma$ is the standard deviation of $\bar{x}$. Equation 7 can be understood informally to mean that, assuming the experimental results are consistent with theory, then the value of $Q_{\text{est}}$, predicted by theory, is likely to lie within the limits defined by Equation 7. This informal interpretation can be made more precise. Specifically, one specifies a confidence interval, e.g. the 95% confidence interval (See below.). Then assuming that the theory accounts for the experimental results, there is a 95% probability that the calculated confidence interval from some future experiment encompasses the theoretical value. If, for a given experiment the value of $x_t$ predicted by theory lies outside of the confidence interval, the assumption that theory accounts for the results of the experiment is rejected, i.e. the experimental results are inconsistent with theory. The confidence interval is expressed as

$$[Q_{\text{est}} - X\ \delta Q, Y_{\text{est}} + X\ \delta Q]\ , \tag{8}$$

The quantity $X$ is obtained from a Student's t-distribution and depends on the confidence interval and the degrees of freedom. A detailed and illuminating discussion of the Student's t-distribution can be found in the Wikipedia on-line free encyclopedia. [1] There are various ways of obtaining or calculating the value of $X$. For example, the spreadsheet Microsoft Excel includes a library function T.INV.2T for calculating $X$ based on a two-tailed t-test. Specifically,

$$X = \text{T.INV.2T}(p, \nu)\ , \tag{9}$$

where the probability $p = 1 - \frac{\text{(the confidence interval)}}{100}$ and $\nu$ is the degrees of freedom. Consider the following example for illustrative purposes. The number of data points is 4; the confidence interval is 95%. Therefore

$$X = \text{T.INV.2T}(1 - \frac{95}{100}, 4 - 1) = 3.182\ , \tag{10}$$

# References

[1] Wikipedia. Student's t-distribution. `https://en.wikipedia.org/wiki/Student%27s_t-distribution`, 2017. [Online; accessed 22-March-2017].

| $D$(m) |
| --- |
| |

Table 1: Datum

| | $I$ (amps) | $\theta_x$ (degrees) | $B_x$ ($\mu$T) | $B_h$ ($\mu$T) |
| --- | --- | --- | --- | --- |
| 1 | | 30 | | |
| 2 | | 45 | | |
| 3 | | 60 | | |
| 4 | | 75 | | |

Table 2: Data and Calculations

| | Experiment | NGDC website |
| --- | --- | --- |
| $\theta_i$ (degrees) | | |
| $B_h$ ($\mu$T) | $\pm$ | |
| $B_e$ ($\mu$T) | | |

Table 3: Results