Diffraction

Introduction

A diffraction grating is a device used for dispersing light in a way that allows its constituent wavelengths to be identified. In this regard it is similar to a prism; however, the manner by which it causes dispersion is, at least superficially, different from that of a prism. Some diffraction gratings, such as the one used in this experiment, consist of a flat piece of glass on which closely spaced lines are etched. Light impinging on one side of the grating exits from the other side with its component wavelengths dispersing in clearly distinguishable directions. An informative and interesting discussion of the diffraction grating, including its history, theory of operation, and how diffraction gratings are made can be found in the WikipediA, the free encyclopedia, The Diffraction Grating. In Fig. 1 is a depiction of light from a monochromatic source, i.e. a source which produces a single wavelength of light, passing through a diffraction grating and being projected onto a screen. A pattern of alternating bright and and dark regions appears on the screen. The midpoints of the bright regions, i.e. where the brightness is a maximum, occur at discrete angles denoted θ_m $(m = 0, \pm 1, \pm 2 \ldots)$, where each angle is measured with respect to the normal to the plane of the diffraction grating. The wavelength λ of the light is related to these angles and the spacing d between consecutive lines of the diffraction grating according to

$$\lambda = \frac{d \sin(\theta_m)}{m} \, . \tag{1}$$

The quantity m is called the order of the maximum. In this experiment the angles θ_m are obtained from the distance x_m which is the separation between m-th order and zeroth order maxima. Specifically,

$$\theta_m = \arctan(x_m/L) \, , \tag{2}$$

where L is the distance the screen is displaced from the diffraction grating (See Fig. 1.).

The purpose of this experiment is to measure specific wavelengths of light emanating from two sources: a helium neon laser and an LED point source.

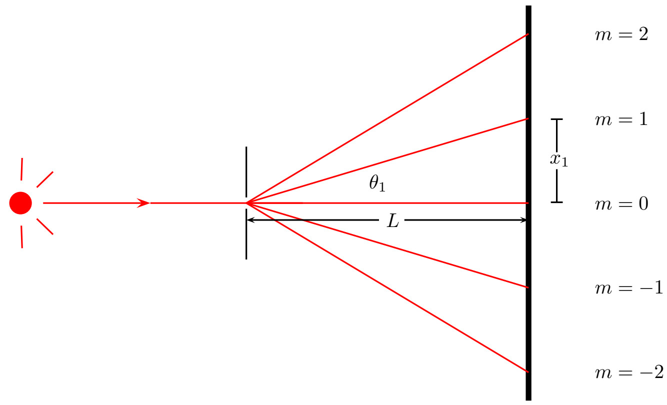

Figure 1: A depiction of the diffraction grating projecting dispersed monochromatic light onto a screen.

Procedure

The Helium Neon Laser

Attach the laser to the optical bench as shown in Fig. 2. The meter stick, which serves as the screen, is attached at the opposite end of the optical bench. Activate the laser and adjust the meter stick so that the light from the laser is centered on the 50 cm mark of the meter stick. Attach the diffraction grating 30 cm from the meter stick, i.e. $L = 30$ cm. Record the positions of the two first-order maxima, i.e. $m = 1$ and $m = -1$, in Table 1. In the Table they are designated $P_{1,l}$ and $P_{1,r}$ depending on whether the measurement is to the left or right of the 50 cm mark on the meter stick. Record the positions of the two second-order maxima, $P_{2,l}$ and $P_{2,r}$. Perform the following calculations and record the results in Table 1.

1. Calculate x_m, $(m = 1, 2)$

$$x_m = |P_{m,r} - P_{m,l}|/2\,. \tag{3}$$

2. Using Eq. 2 calculate θ_1 and θ_2.

3. For each value of θ_m calculate a value for λ using Eq. 2. The line separation $d = 1.667 \times 10^3$ nm.

4. Report the value of λ in Table 1, in accord with Eq. 7.

5. Obtain the accepted value of λ from the Wikipedia website, Helium–neon laser. Using the criterion given in Eq's 8, 9, and 10, report the 95% confidence interval for λ in Table 2. Is the value of λ obtained in the experiment consistent with the accepted value at the 95% confidence level?

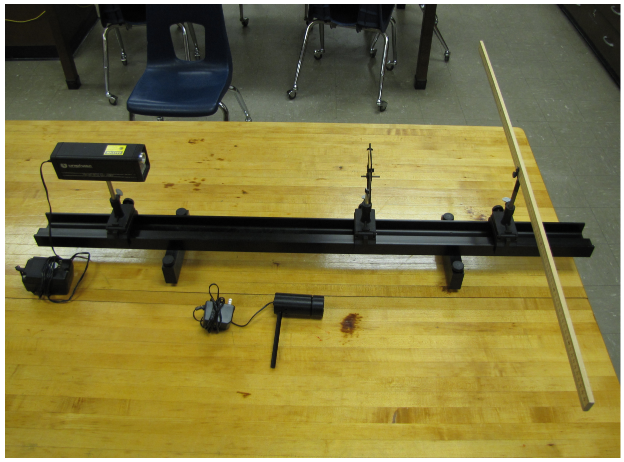

Figure 2: Positioned on the optical bench is the laser, the diffraction grating, and meter stick used for displaying the diffraction pattern. In the forground is the LED point source used for producing the white light in the second part of the experiment.

6. Using the accepted value for the wavelength of light produced by the helium–neon laser and the line separation for the diffraction grating used in this experiment, can you calculate the angle θ_3 corresponding to the $m = 3$ maximum of the diffraction grating? If so, what is its value? If not, why are you unable to perform the calculation?

The LED Point Source

Remove the laser from the optical bench and attach the LED point source. The light from the lamp is white implying that the light comprises a broad range of wavelenghts in the visible spectrum. In the first order maximum, i.e. $m = 1$, notice the colors ranging from red to violet. You will determine the range of wavelengths constituting the visible spectrum of light. Specifically, you will determine the extreme wavelengths of red and violet light that are visible. The visible spectrum comprises all wavelengths between these extremes.

1. In Table 3 record the positions of the two first-order maxima $P_{1,l}$ and $P_{1,r}$ of the extreme wavelength of red light. The spectrum gradually dims beyond this point. Do the same for the violet light.

2. Using Eq's 3, 2, and 1 calculate the wavelengths of the red and violet light. Record their values in Table 3.

Appendix

Given a set of data x_i ($i = 1 \ldots N$) corresponding to a quantity whose true value is x_t. If each of the x_i differs from x_t because each x_i includes a random error ϵ_i, i.e. $x_i = x_t + \epsilon_i$, then an unbiased estimate of x_t is $\bar{x}$,

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i \; , \tag{4}$$

and an unbiased estimate of its standard error is σ,

$$\sigma = \frac{\sigma_{N-1}}{\sqrt{N}} \; , \tag{5}$$

where

$$\sigma_{N-1} = \sqrt{\frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{N-1}} \; . \tag{6}$$

In calculating σ_{N-1}, the number of degress of freedom $\nu = N - 1$ is used rather than N. Note: In Microsoft Excel $\bar{x}$ and σ_{N-1} can be calculated using the library functions AVERAGE and STDEV.

To reflect the statistical uncertainty in x_t, the experimental results are typically reported as

$$Q_{\text{est}} \pm \delta Q \; , \tag{7}$$

where $Q_{\text{est}} = \bar{x}$ is the unbiased estimate of x_t and $\delta Q = \sigma$ is the standard deviation of $\bar{x}$. Equation 7 can be understood informally to mean that, assuming the experimental results are consistent with theory, then the value of Q_{est}, predicted by theory, is likely to lie within the limits defined by Equation 7. This informal interpretation can be made more precise. Specifically, one specifies a confidence interval, e.g. the 95% confidence interval (See below.). Then assuming that the theory accounts for the experimental results, there is a 95% probability that the calculated confidence interval from some future experiment encompasses the theoretical value. If, for a given experiment the value of x_t predicted by theory lies outside of the confidence interval, the assumption that theory accounts for the results of the experiment is rejected, i.e. the experimental results are inconsistent with theory. The confidence interval is expressed as

$$[Q_{\text{est}} - X\ \delta Q, Q_{\text{est}} + X\ \delta Q] \; , \tag{8}$$

The quantity X is obtained from a Student's t-distribution and depends on the confidence interval and the degrees of freedom. A detailed and illuminating discussion of the Student's t-distribution can be found in the Wikipedia on-line free encyclopedia. [1] There are various ways of obtaining or calculating the value of X. For example, the spreadsheet Microsoft Excel includes a library function T.INV.2T for calculating X based on a two-tailed t-test. Specifically,

$$X = \text{T.INV.2T}(p, \nu) \; , \tag{9}$$

where the probability $p = 1 - \frac{\text{(the confidence interval)}}{100}$ and ν is the degrees of freedom. Consider the following example for illustrative purposes. The number of data points is 2; the confidence interval is 95%. Therefore

$$X = \text{T.INV.2T}(1 - \frac{95}{100}, 2 - 1) = 12.706\,. \tag{10}$$

References

[1] Wikipedia. Student's t-distribution. https://en.wikipedia.org/wiki/Student%27s_t-distribution, 2017. [Online; accessed 22-March-2017].

m	$P_{m,l}$ (cm)	$P_{m,r}$ (cm)	x_m (cm)	θ_m (degrees)	λ (nm)
1					
2					
					$\pm$

Table 1: The Helium Neon Laser – Data and Calculations

$\bar{\lambda} - X\sigma$	$\bar{\lambda} + X\sigma$

Table 2: The 95% Confidence Interval

Case	$P_{1,l}$ (cm)	$P_{1,r}$ (cm)	x_1 (cm)	θ_1 (degrees)	λ (nm)
red					
violet					

Table 3: The LED Point Source – Data and Calculations