

Almost Elastic Collisions in One Dimension

Introduction

In this experiment a system comprising two carts of masses, m_1 and m_2 , constrained to move in one dimension, collide. In order to minimize the effects of external forces on the system, such as friction, gravity, or the normal force of the track on the carts, the experiment is performed on an air track adjusted so that the carts are restricted to move perpendicular to the force of gravity. Using the air track helps to minimize the effect of kinetic friction because as the carts move they are supported on an air cushion. By restricting the carts to move perpendicular to the force of gravity the normal force of the track on the carts is equal in magnitude, but opposite in direction to the force of gravity on the carts. Thus, the net force, either tangential or perpendicular to the track is zero, justifying the assumption that the momentum of the system is conserved. During the collision itself the force between the two carts results from contact between two flexible metal bands which compress during the collision process in an approximately elastic manner. This interaction helps to minimize the amount of kinetic energy dissipated during the collision process, justifying the assumption that kinetic energy is conserved during the collision, i.e. that the collision is elastic.[1] In the experiment cart number two is initially at rest. Then, as a consequence of momentum and kinetic energy conservation, velocities V_1 and V_2 of carts one and two after the collision are related to the velocity, v_1 , of cart one before the collision, by the relationships¹

$$\frac{1}{V_1} = \left(\frac{m_1 + m_2}{m_1 - m_2} \right) \frac{1}{v_1}, \quad (1)$$

$$\frac{1}{V_2} = \left(\frac{m_1 + m_2}{2m_1} \right) \frac{1}{v_1}. \quad (2)$$

Although the experiment has been designed to satisfy the conditions of energy and momentum conservation, in reality, there will be some violation of these assumptions because of some energy dissipating effects or some non-vanishing, net external force acting on the system. The purpose of this experiment is threefold: to make precise measurements describing the system before and after the collision, to analyze the data using Eqs. 1 and 2, and finally to test the extent to which the assumption of either momentum or kinetic energy conservation is violated.

¹The relationships in the velocities have been expressed in this atypical way to facilitate the ensuing statistical analysis.

Procedure

The equipment employed in performing this experiment is shown in Figure 1 (Note: the function setting on the digital timer should be set to S1.). Measurements obtained using the equipment are accurate to four significant figures. Consequently, not only should data be recorded to this degree of accuracy, but all computations should also be performed to four significant figures. Perform the experiment and analysis as follows.

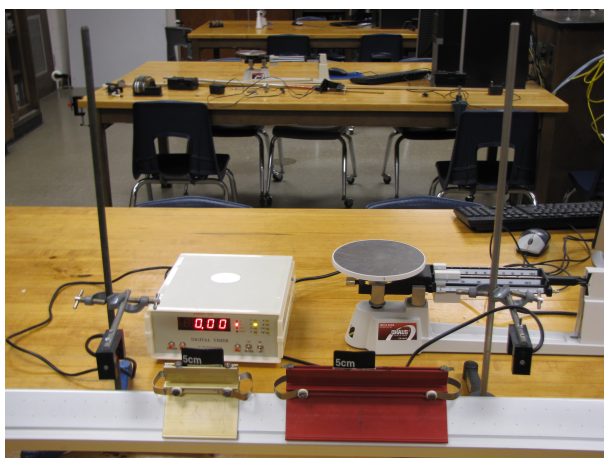


Figure 1: Momentum Apparatus

1. first, level the air track. Place the gold cart on the air track, near its center. Turn on the air supply. Locate the two leveling knobs on the base supporting one end of the air track. Adjust the two knobs until the cart remains stationary.
2. Measure the masses of cart one and cart two. Cart one is the more massive of the two. Report the results in Table 1.
3. Measure the widths of the flags protruding from the top of each cart.
4. Perform five trials, each consisting of propelling cart one so that it collides with cart two, which is initially at rest. Select a large range of speeds, but not so large as to disrupt the operation of the equipment. Obtain the duration of time for cart one to pass through the photogate before the collision, as well as the durations of time for carts one and two to pass through the second photogate after the collision.
5. Calculate the reciprocal of the speeds of cart one, before the collision and carts one and two, after the collision. Report the results in Table 2.
6. Using Excel plot two sets of data on the same graph: reciprocal speed of cart one after collision (ordinate) vs. reciprocal speed of cart one before the collision (ab-

scissa) and reciprocal speed of cart two after collision (ordinate) vs. reciprocal speed of cart one before the collision (abscissa).

7. Perform a linear regression on each set of data, forcing the intercept to be zero for each equation estimated. Report each value of R^2 and each regression equation on the graph.
8. From Equations 1 and 2, calculate the theoretical value of each slope. Using Eq. 7 calculate the standard error of the slope, δM . Report the theoretical value of the slope and the estimated value of the slope, along with its standard error, in Table 3.
9. Calculate the 95% confidence interval using Equation 9. Report the values in Table 3. Are theory and experiment consistent at the 95% confidence interval? What do you conclude regarding the assumption that the collision is elastic?

Appendix

Consider two sets of data y_i and x_i ($i = 1 \dots N$) where the y_i are assumed to be linearly related to the x_i according to the relationship

$$y_i = Mx_i + B + \epsilon_i. \quad (3)$$

The ϵ_i is a set of uncorrelated random errors. Estimates of the slope M_{est} and y-intercept B_{est} can be obtained using a variety of techniques, such as the method of least squares estimation. The degrees of freedom $\nu = N - 2$, since the number of parameters being estimated is 2, i.e. M and B . It can be shown that the standard error of the slope δM is

$$\delta M = \sqrt{\frac{1 - r^2}{N - 2}} \frac{\sigma_y}{\sigma_x}, \quad (4)$$

and the standard error of the intercept δB is

$$\delta B = \sigma_y \sqrt{\frac{1 - r^2}{N - 2}} \sqrt{\frac{N - 1}{N} + \frac{\bar{x}^2}{\sigma_x^2}}, \quad (5)$$

where r is the estimated correlation coefficient between the y_i and x_i , σ_y and σ_x are the estimated standard deviations of the y_i and x_i , and \bar{x} is the estimated average of the x_i .² The quantities \bar{x} , σ_x , and σ_y can straightforwardly be calculated using function keys on a scientific calculator or defined functions in Excel. If, in the regression equation, Eq. 3, the quantity B , the y-intercept, is required to be zero, i.e. the regression equation is now

$$y_i = Mx_i + \epsilon_i, \quad (6)$$

then the degrees of freedom $\nu = N - 1$ so that

$$\delta M = \sqrt{\frac{1 - r^2}{N - 1}} \frac{\sigma_y}{\sigma_x}. \quad (7)$$

²Note: in Equation 5 σ_x and σ_y are unbiased estimates.

To reflect the statistical uncertainty in a quantity Q , where Q is either the slope M or y-intercept B , the quantity Q is typically reported as

$$Q_{\text{est}} \pm \delta Q , \quad (8)$$

which can be understood informally to mean that, assuming the experimental results are consistent with theory, then the value of Q , predicted by theory, is likely to lie within the limits defined by Equation 8. This informal interpretation can be made more precise. Specifically, one specifies a confidence interval, e.g. the 95% confidence interval (See below.). Then assuming that the theory accounts for the experimental results, there is a 95% probability that the calculated confidence interval from some future experiment encompasses the theoretical value. If, for a given experiment the theoretical value Q lies outside of the confidence interval, the assumption that theory accounts for the results of the experiment is rejected, i.e. the experimental results are inconsistent with theory. The confidence interval is expressed as

$$[Q_{\text{est}} - X \delta Q, Q_{\text{est}} + X \delta Q] , \quad (9)$$

The quantity X is obtained from a Student's t-distribution and depends on the confidence interval and the degrees of freedom. A detailed and illuminating discussion of the Student's t-distribution can be found in the Wikipedia on-line free encyclopedia. [2] There are various ways of obtaining or calculating the value of X . For example, the spreadsheet Microsoft Excel includes a library function T.INV.2T for calculating X based on a two-tailed t-test. Specifically,

$$X = \text{T.INV.2T}(p, \nu) , \quad (10)$$

where the probability $p = 1 - \frac{(\text{the confidence interval})}{100}$ and ν is the degrees of freedom. Consider the following example for illustrative purposes. The number of data points is 5; the confidence interval is 95%. The data are assumed to be linearly related with an intercept of zero so that Equation 6 applies. Therefore

$$X = \text{T.INV.2T}(1 - \frac{95}{100}, 5 - 1) = 2.776 , \quad (11)$$

References

- [1] Wikipedia. Elastic collision — Wikipedia, the free encyclopedia. http://en.wikipedia.org/wiki/Elastic_collision, 2007. [Online; accessed 14-October-2007].
- [2] Wikipedia. Student's t-distribution. https://en.wikipedia.org/wiki/Student%27s_t-distribution, 2017. [Online; accessed 22-March-2017].

	Cart One	Cart Two
Mass (gm)		
Width of Flag (mm)		

Table 1: Data

Before Collision			After Collision			
Cart One			Cart One		Cart Two	
time(ms)	speed ⁻¹ (ms/mm)		time(ms)	speed ⁻¹ (ms/mm)	time(ms)	speed ⁻¹ (ms/mm)
1						
2						
3						
4						
5						

Table 2: Data and Calculations I

	Cart One	Cart Two
Slope(exp) ($M_{\text{est}} \pm \delta M$)		
95% confidence ($M_{\text{est}} - X\delta M$) ($M_{\text{est}} + X\delta M$)		
Slope(theory)		
Are theory and experiment consistent? (y or n)		

Table 3: Calculations II