A Strategy for Trading the S&P 500 Futures Market

Edward Olszewski*

Abstract

A system for trading the S&P 500 futures market is proposed. The system is applied to S&P 500 futures data during the period from September 14, 1987, to September 27, 1999. The system uses a momentum oscillator for generating entry or exit prices. In addition, the system uses another indicator for predicting the direction of the trend. When only the oscillator is used for selecting trades, the system is not, in general, as good as buy-and-hold. However, when the trend indicator is used as a filter, the trading system is, at least, as good as buy-and-hold. (*JEL* G14)

Introduction

Results of a previous study (EAO) suggested that inefficiency might have existed in some futures markets (Olszewski 1998). In that study a trend-following trading system was applied to ten futures markets and shown to generate large, hypothetical profits for a few diverse markets; however, for some of the markets considered, notably the S&P 500, the trading system was unable to generate profits consistently. In that study the reason speculated for this failure in the case of the S&P 500 was that a more appropriate trading system might be one which generates buy or sell signals counter to the direction of market momentum, i.e., buy on "high negative momentum" or sell on "high positive momentum." The purpose of this study is to evaluate the profitability of trading the S&P 500 futures index using such a trading system. Evaluating the profitability of a trading system requires that some benchmark be selected for comparison. For this purpose the strategy buy-and-hold (BAH) has been chosen. The reason for this choice is that BAH has been and continues to be a relatively profitable strategy since the early 1980s. The trading system in this study is based on principles similar to those on which the trading system in the EAO study is based, utilizing such information as momentum and trend characteristics of prices. However, the trading system in this study differs from that in the EAO study because it incorporates an additional filter for trade selection based on the commitment of traders report released biweekly by the Commodity Futures Trading Commission (CFTC).

The organization of the paper is as follows. In the second section, data used in this study are presented, and the procedure for constructing a continuous price series is explained. Next, a

*Edward Olszewski, Department of Physics, University of North Carolina at Wilmington, Wilmington, NC 28403, moof@pootie.phy.uncwil.edu.

rigorous definition of market momentum derivable from the path integral representation of the Langevin equations is introduced. Long-range memory processes are then briefly described, and the modified rescaled range (R/S) statistic of Lo is presented (Lo 1991) Finally, this section concludes with the introduction and interpretation of a descriptive statistic calculated from data in the commitment of traders report. In the third section, the long-range dependence of the S&P 500 futures data is assessed using the R/S statistic. Then a trading system based on market momentum is proposed. The trading system is applied to the S&P 500 futures data and its profitability evaluated. The trading system is modified using the descriptive statistic to filter trades, and the profitability of the trading system is re-evaluated. Finally, there is discussion about how performance of the modified trading system compares with buy-and-hold. In the fourth section, results are summarized, and a future direction of study is outlined.

Data and Methodology

Data

The time frame within which the trading system is applied covers the period from 9/14/87 to 9/27/99. This time frame contains 3,352 data points. The data set is divided into two subsets: the first from 9/14/87 to 6/9/93 (1,451 trading days) and the second from 6/10/93 to 9/27/99 (1,583 trading days). The trading system is applied to each subset separately. There are two reasons for dividing the data set. The time frame of the first subset corresponds to the time frame used in the EAO study so that comparisons can be made with that study. Also, division of the dataset into two subsets permits changes in the profitability of the trading system to be observed over time. The data consist of daily high, low, and settlement prices of the S&P 500 futures index for the delivery months March, June, September, and December. Also, 1,316 additional data points, prior to 6/14/87, are prepended to the data for initializing parameters in the trading system. Thus, the total number of data points is 4,352, beginning on 6/30/82 and ending on 9/27/99.

On any given day multiple futures contracts trade; however, a single time series must be constructed for the subsequent analysis. The method selected for constructing the time series has the advantage of mimicking the daily percent change in price of a trade, which is continually rolled over from the near contract to the first forward contract. The trade is rolled over at the conclusion of trading on the last trading day of the month prior to the expiration month of the near contract. The procedure for constructing the bivariate series (S_t, r_t) proceeds as in EAO.

First, on a given day t for all contracts, the quantities s_t and r_t are calculated from

$$s_t = \ln(P_t) - \ln(P_{t-1}) \quad , \tag{1}$$

and

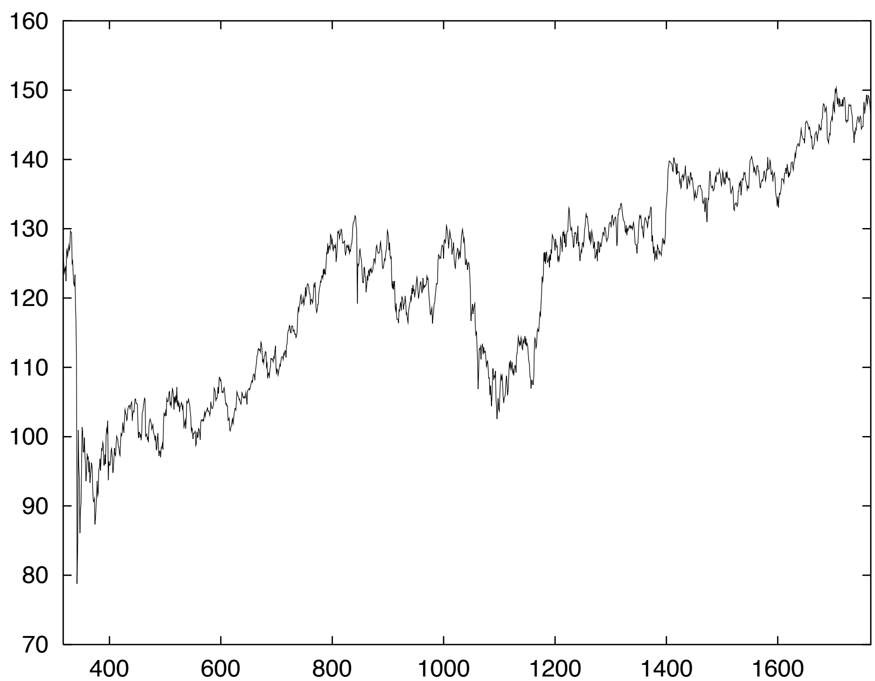
$$r_t = \max\begin{cases} \ln(H_t) - \ln(L_{t-1}) \\ |\ln(H_t) - \ln(P_{t-1})| \\ |\ln(L_t) - \ln(P_{t-1})| \end{cases} , \tag{2}$$

where (H_t, L_t, P_t) are the high, low, and settlement prices of a given contract. A single bivariate time series is obtained by selecting consecutive pairs (s_t, r_t) from the contract beginning with the earliest expiration date and continuing until the last trading day of the month prior to the expiration month of that contract. On that day pairs are then selected from the first forward contract. The procedure is continued in this manner until a single pair (s_t, r_t) is obtained for each trading day. This procedure provides a consistent way of selecting a single pair (s_t, r_t) out of all such pairs from the different futures contracts trading on a given day. The continuous bivariate, time series (S_t, r_t) used in the analysis is then defined by:

$$S_t = \begin{cases} 100 & t = 0 \\ S_{t-1}\exp(s_t) & otherwise \end{cases}, \tag{3}$$

The series S_t is arbitrarily defined so that $S_0 = 100$, where $t = 0$ corresponds to June 13, 1986. The first component of the time series, S_t, is a synthetic price series reflecting percent change in the value of the futures contract. This series offers the advantage of being devoid of gaps in price, which typically occur when prices are rolled over from the near contract to the first forward contract. The second component, r_t, is a measure of daily market volatility.

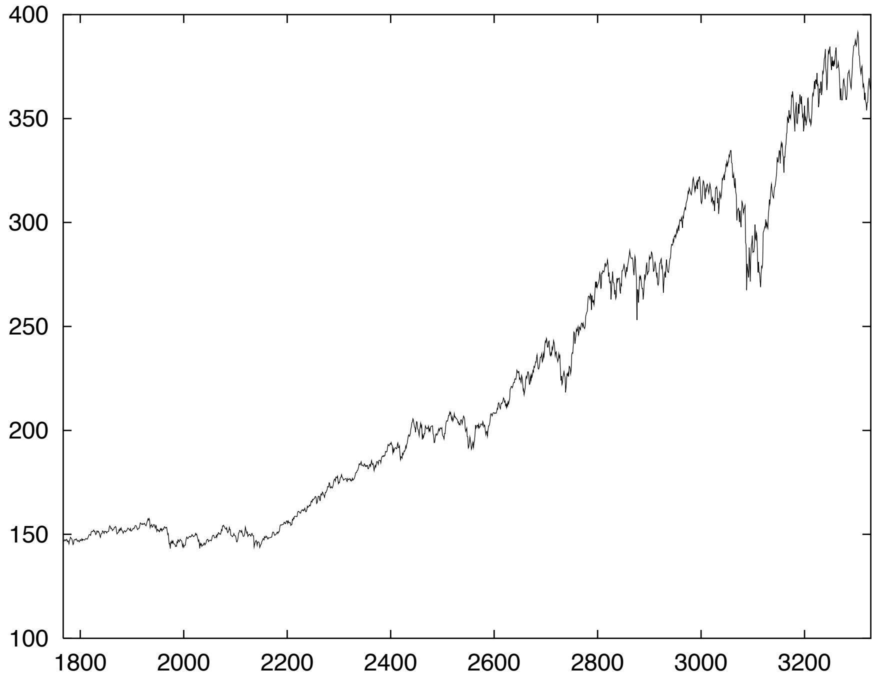
FIGURE 1. CONTINUATION CONTRACT OF DAILY SETTLEMENT PRICES FOR THE S&P 500 INDEX FROM SEPTEMBER 14, 1987 TO JUNE 8, 1993



Notes: The value of the contract on June 13, 1986, has been arbitrarily set to 100.

Figures 1 and 2 show the synthetic price series, S_t, for the time-span under consideration. From Figure1, which covers about six years, it can be seen that the index has appreciated approximately 15.0 percent, representing 2.45 percent compounded, annual return. The fact that the plot begins about one month prior to the crash of 1987 contributes appreciably to the modest rate of return. Figure 2 also spans a time of approximately six years. In this case the rate of return is greater. Over this span of time the price increases approximately 255 percent, resulting in a compounded, annual return of 17.0 percent.

FIGURE 2. CONTINUATION CONTRACT OF DAILY SETTLEMENT PRICES FOR THE S&P 500 INDEX FROM JUNE 9, 1993, TO SEPTEMBER 27, 1999



Momentum

The trading system to be tested utilizes a momentum indicator for entering and exiting positions. The momentum indicator can be derived rigorously from theoretical considerations (see EAO and references therein). Its derivation is now outlined here. The derivation begins with a set of first-order, stochastic differential equations, specifically Langevin equations, whose solution is $q(t)$, t being a continuous time parameter. Alternatively, the Langevin equations can be represented as a path integral which is the conditional probability density of the $q(t)$. A Lagrangian can be obtained from the path integral, and the momentum can be rigorously defined as an appropriate derivative of the Lagrangian. The specific discrete version of the stochastic equation relevant for the subsequent analysis is

$$q_t - \mu = \sigma\varepsilon_t \quad , \tag{4}$$

where t is an integer. Here σ and μ are parameters characterizing q_t. In addition, ε_t is zero mean and unit variance white noise. It can be shown that the momentum corresponding to q_t in Equation 4 is given by

$$p_t = \frac{q_t - \mu}{\sigma^2} \quad . \tag{5}$$

Long-Range Memory Processes

The hydrologist H. E. Hurst pioneered the study of long-range memory processes in his investigation of the storage capacity of reservoirs (Peters 1994; Hurst 1951). Extending Hurst's work, Mandelbrot and others introduced a family of Gaussian random functions, designated fractional Brownian motions (fBm’s), which exhibit long-term persistence similar to those studied by Hurst (Mandelbrot and Van Ness 1968; Mandelbrot and Wallis 1969a-c; Beran 1994). As described in EAO, fBm’s possess a number of interesting properties. The spectral densities of fBm’s are proportional to $f^{\,1-2H}$, f being the frequency and H, $0 < H < 1$, being a parameter characterizing the fBm.[1] They exhibit the following scaling behavior in the variance $\sigma^2(t) = t^{2H}\sigma^2(1)$, where $\sigma(t)$ is the standard deviation of the fBm at time t. As detailed in Mandelbrot and Wallis for $H > .5$, the future and past are positively correlated, with correlations approaching 1 as H approaches 1. These fBm’s exhibit a long-range memory characterized by a persistence in positive correlation between events which are increasingly separated in time. Consequently, their autocorrelations decay much more slowly than short-range memory processes like autoregressive or moving average. For $H = .5$ the future and past are uncorrelated. For $H < .5$ the future and past are negatively correlated, with correlations decreasing to -.5. These fBm’s are characterized by anti-persistence, i.e., correlations are all negative. Such series are subject to more reversals than Brownian motion.

Detecting long-range memory dependence in time series, such as is found in fBm’s, can be complicated by the presence of short-range memory dependence. Indeed, it has been pointed out by Lo that the classical R/S analysis used to identify long-range memory dependence can falsely indicate its presence in a time series when, in fact, only short-range memory dependence is present. Lo has derived a test statistic for detecting long-range memory dependence similar to the classical R/S statistic, R_n (Lo 1991). It is less prone to the misspecification of classical R/S analysis, while also exhibiting power against rejecting long-range memory dependence when it is present. The test statistic takes account of short-range memory dependence by adjusting the variance used in R_n. Specifically, given n consecutive points from a discrete time series $\{x_t\}$, the test statistic Q_n is defined as[2]

$$Q_n \equiv \frac{\tilde{R}_n}{\sigma_n(q)} \ , \tag{6}$$

where

$$\tilde{R}_n = \max_{1\le k\le n}(x_1 + \ldots + x_k - k\bar{x}) - \min_{1\le k\le n}(x_1 + \ldots + x_k - k\bar{x}) \ , \tag{7}$$

$$\bar{x} = \frac{1}{n}\sum_{k=1}^{n} x_k \ , \tag{8}$$

$$\sigma_n^2(q) = s_n^2 + \frac{2}{n}\sum_{j=1}^{q}\omega_j(q)\left\{\sum_{i=j+1}^{n}(x_i - \bar{x})(x_{i-j} - \bar{x})\right\} \ , \tag{9}$$

[1] The case $H = .5$ corresponds to ordinary Brownian motion.

[2] The x_t corresponds to the s_t given in Equation 1.

$$s_n = \sqrt{\frac{\sum_{k=1}^{n}(x_k - \bar{x})^2}{n}} \quad , \tag{10}$$

and

$$\omega_j(\tilde{q}) = 1 - \frac{j}{\tilde{q}+1} \quad . \tag{11}$$

The value of the truncation lag, $\tilde{q}$, depends on the data being considered. Its value must be large enough to account for short-range memory dependence in the data, but not so large as to alter the finite sample distribution of Q_n radically. Andrews suggests the following data dependent rule for selecting $\tilde{q}$ (Lo 1991; Andrews 1991):

$$\tilde{q} = [k_n] \ , \quad k_n = \left(\frac{3n}{2}\right)^{\frac{1}{3}}\left(\frac{2\hat{\rho}}{1-\hat{\rho}^2}\right)^{\frac{2}{3}} \quad , \tag{12}$$

where $[k_n]$ is the greatest integer less than or equal to k_n and $\hat{\rho}$ is the estimated first-order correlation of the data. Also, the weights $\omega_j(\tilde{q})$ in Equation 11 are replaced by

$$\omega_j(\tilde{q}) = 1 - \left|\frac{j}{k_n}\right| \quad . \tag{13}$$

Lo defines the statistic $V_n(\tilde{q})$, which is based on the test statistic Q_n,

$$V_n(\tilde{q}) \equiv \frac{Q_n}{\sqrt{n}} \quad . \tag{14}$$

This statistic is identical to the statistic V_{nj} used by Hurst and Peters, except that the variance has been adjusted to account for short-range memory effects (Peters 1994). Lo derives the limiting distribution of V_n for which $V_n(\tilde{q})$ is an estimator, under the assumption of no long-range memory dependence and a general set of conditions which include strong mixing (short-range memory) and conditional heteroskedasticity. In Table 1 the fractiles of the limiting distribution of the V statistic are presented. Small values of V correspond to anti-persistence, and large values correspond to persistence. If $V_n(\tilde{q})$ is less than .861, the hypothesis of anti-persistence at the 5 percent confidence level would be accepted. Similarly, if $V_n(\tilde{q})$is greater than 1.747, the hypothesis of persistence at the 5 percent confidence level would be accepted.

Commitment of Traders Report

The "futures only" component of the commitment of traders report is released every two weeks, usually on Friday, at 3:30 p.m. eastern time by the CFTC (the commitment of traders report). The report includes extensive information about the open interest of futures markets in which five or more traders hold positions equal to or in excess of the limits established by the CFTC. Each report contains information about open interest of the previous two Tuesdays. The open interest is classified as reportable and non-reportable. Non-reportable positions are those traders' positions

TABLE 1. FRACTILES OF THE LIMITING DISTRIBUTION OF THE V STATISTIC UNDER THE ASSUMPTION OF NO LONG-RANGE MEMORY

Prob($V < v$)	.005	.0250	.050	.100	.200	.300	.400	.500
v	0.721	0.809	0.861	0.927	1.018	1.090	1.157	1.223
Prob($V < v$)	.543	.600	.700	.800	.900	.950	.975	.995
v	$\sqrt{\frac{\pi}{2}}$	1.294	1.374	1.473	1.620	1.747	1.862	2.098

that are below the limits established by the CFTC. Of interest in this study are the reportable positions, which are positions held by traders in excess of limits established by the CFTC. That limit for the S&P 500 futures market is 600 contracts. The reportable positions are subdivided into commercial (traders who are classified as reportable and engage in hedging) and non-commercial. For the purpose of this study the positions held by commercials are important. The specific information needed from the report is the number of long positions, N_{lo}, and the number of short positions, N_{sh}, held by commercials. From these quantities the following descriptive statistic is defined

$$I = \frac{N_{lo} - N_{sh}}{N_{lo} + N_{sh}} \quad . \tag{15}$$

The statistic I, $(-1 \leq I \leq +1)$, measures the fraction of total open interest held by hedgers, i.e., commercials. Whenever $I > 0$, hedgers are net long; for $I < 0$, hedgers are net short. If $I = 0$, the number of long and short hedged positions is equal.

Trading System

A trading system consistent with the following paradigm of market behavior is proposed. While trending either up or down, the market may experience either an over-bought or over-sold condition. Buy (sell) signals are generated when the market becomes over-sold (over-bought). Typically, normalized momentum indicators (oscillators) can be used to determine whether a market is over-bought or over-sold. Some examples of normalized momentum indicators are Wilder's RSI, Lane's Stochastics, or Lambert's CCI. Murphy discusses the use of such oscillators in this context (Murphy 1986, p. 279-284). Momentum, as defined by Equation 5, is another example of a normalized momentum indicator and constitutes one component of the trading system used in this study. According to Equation 5, momentum is obtained by subtracting the conditional mean from the price and dividing the result by the conditional variance. The definition is intuitively appealing because momentum is based not only on how much the price deviates from what is expected but also on how much it deviates in comparison to current volatility. The drawback of this definition is that one needs estimates of the conditional mean, i.e., what is expected, and also the conditional variance, i.e., current volatility, to calculate its value.

The trading system uses momentum in conjunction with the predicted direction of the trend. Specifically, buy signals are taken only when the predicted direction of the trend is up, and sell signals only when it is down. It is assumed that the direction of the trend can be predicted from the net open interest of hedgers. If hedgers are net long (short), the trend is predicted to be up (down).

The hedge statistic I, Equation 15, is used for this purpose.[3] The assumption is debatable since it is contrary to the idea that speculators, on average, earn profits by assuming the risk of hedgers. The obvious implication is that speculators would, on average, lose money, which may imply a negative risk premium. Whether this assumption can be justified rests, in part, on whether the trading system is profitable.

Persistent vs. Anti-Persistent Behavior and the Modified R/S Test

It is now shown that the S&P 500 futures index has the tendency to exhibit anti-persistent behavior. Such behavior justifies using momentum to determine over-bought or over-sold conditions. If prices are anti-persistent, then large price moves in a given direction are likely to be followed by price moves in the opposite direction. Thus, the presence of anti-persistence in price data is compatible with the idea of using momentum to determine whether a market is over-bought or over-sold. The R/S statistic $V_n(q)$ given in Equation 14 can be used for this purpose. To test whether the S&P 500 data is anti-persistent, the R/S statistic has been calculated for 46 consecutive, overlapping subsamples of data, each subsample comprising 1,315 points. Each successive subsample ends 66 trading days in the future of the previous subsample. To perform this subdivision an additional 1,000 data points have been prepended to the data set, resulting in the following subsamples [1, 1,315], [67, 1,381], [133, 1,447], [199, 1,513], ..., [2,905, 4,219].[4]

The statistic $V_n(q)$ has been calculated for each subsample using Equations 12, 13, and14. The reason for selecting subsamples of 1,315 points, i.e., $n = 1{,}315$, is based in part on Lo's study of the finite sample properties of $V_n(q)$ in which he has simulated data possessing various long-range or short-range dependencies[6]. In these studies Lo has used sample sizes (n=100, 250, 500, 750, 1,000). Lo has found that for $n = 1{,}000$ asymptotic properties of the V statistic are reasonably well approximated.[5] In Table 2 the R/S statistic $V_{1315}(q)$ is displayed for the various subsamples. For contrast and comparison the R/S statistic of the Japanese *yen* is also displayed, since it is generally believed that the currencies exhibit strong trending characteristics and thus should be persistent (see Olszewski 1998 study of EAO). According to Table 1, large values of V, ($V > 1.223$) correspond to persistence and small values, i.e., ($V < 1.223$), to anti-persistence. Thus, it is apparent from Table 2 that the S&P 500 futures prices have consistently exhibited anti-persistent behavior. This provides some justification for testing the counter-to-momentum (C2M) trading system described in the next section. On the other hand, the results of Table 2 suggest that, in the case of the Japanese *yen*, a trend-following trading system may be more appropriate. Indeed, in the study of EAO a trend-following trading system was applied to the *yen* and found to be profitable during the time period covered by the first subset.

Momentum-Based Trading System

To operationalize the previously presented paradigm of market behavior, it is assumed that prices fluctuate about some approximate equilibrium value. This simplifies calculations and reduces the computational time for performing them.[6] This assumption implies the following

[3] The data for calculating I typically become available every two weeks, on Friday afternoon. Included in the data is information about open interest for each of the two previous Tuesdays. The value of I calculated is based on the data of the previous Tuesday. Since the data are available so late in the day, the value of I is not used for trading until the following Monday.

[4] These subsamples have been selected because each one ends one day before the beginning of a 66-day period used in testing the trading system. See "Momentum-Based Trading System."

[5] Other than this motivation the selection of $n = 1{,}315$ has been an ad hoc decision.

[6] More complicated assumptions have been tried with no improvement in the perfomance of the trading system.

TABLE 2. $V_N(Q)$ STATISTIC PRIOR TO EACH OUT-OF-SAMPLE TRADING PERIOD

Subset One			Subset Two		
Period	S&P 500	Japanese *Yen*	Period	S&P 500	Japanese *Yen*
1,315	1.19	1.81	2,767	1.00	1.51
1,381	0.96	2.02	2,833	1.03	1.49
1,447	0.96	1.98	2,899	1.06	1.49
1,513	0.96	1.96	2,965	1.06	1.52
1,579	0.95	2.02	3,031	1.00	1.22
1,645	0.95	1.91	3,097	1.02	1.02
1,711	0.96	2.01	3,163	1.13	1.07
1,777	0.97	2.16	3,229	0.92	0.87
1,843	0.97	2.00	3,295	0.96	1.01
1,909	0.96	2.02	3,361	0.81	1.13
1,975	0.95	2.00	3,427	1.28	1.70
2,041	0.95	2.00	3,493	1.10	1.77
2,107	0.92	2.12	3,559	1.09	1.69
2,173	0.92	1.86	3,625	1.08	1.79
2,239	0.92	1.58	3,691	1.36	2.03
2,305	0.92	1.60	3,757	1.14	2.09
2,371	0.93	1.51	3,823	1.28	2.05
2,437	0.94	1.63	3,889	1.21	1.87
2,503	0.93	1.53	3,955	1.04	2.01
2,569	0.95	1.52	4,021	1.13	1.82
2,635	0.90	1.52	4,087	1.36	1.65
2,701	0.98	1.44	4,153	0.86	1.87
			4,219	0.77	1.86
			4,285	0.77	1.71

model of market prices (Equation 4)

$$q_i = \mu + \sigma\varepsilon_i \quad . \tag{16}$$

Here $q_i = \ln(S_i)$, where S_i is given by Equation 3, and $i = t, t{-}1, t{-}2, \ldots, t{-}(n_d - 1)$.[7] The quantity μ corresponds to the equilibrium price, and ε_i is the white noise causing the fluctuations. The quantity μ is estimated by

$$\mu = \frac{\sum_{t-(n_d-1)}^{i=t} q_i}{n_d} \quad . \tag{17}$$

If it is assumed that on any given day i the prices during that day may be approximated by a random walk about its mean, then σ_i may be estimated using the result of Parkinson (Parkinson 1980),

$$\sigma_i^2 = \left(\frac{\pi}{8}\right)^{\frac{1}{2}} E[r_i] = \left(\frac{\pi}{8}\right)^{\frac{1}{2}} r_i \tag{18}$$

Here r_i is given in Equation 2, $E[\ ]$ is expected value, and $E[r_i]$ has been approximated by its best estimate r_i. The quantity σ is estimated by averaging over all σ_i

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[7] It should be noted that the logarithm of price, q_i , is used rather than price, S_i.

$$\sigma = \frac{\sum_{t-(n_d-1)}^{i=t} \sigma_i}{n_d} \quad . \tag{19}$$

The momentum p_i is given by Equation 5. A proxy for the momentum, which is dimensionless, is used in the subsequent analysis. It is given by

$$\tilde{p}_i = p_i \sigma = \frac{q_i - \mu}{\sigma} \quad . \tag{20}$$

The momentum-based component of the trading system can now be defined. There are five adjustable parameters: n_d, n_l, n_s, b_l, b_s, subject to the constraint $n_s \leq n_l < n_d$. There are two average momentum variables,

$$p_t^l = \frac{\sum_{t-(n_l-1)}^{i=t} \tilde{p}_i}{n_l} \tag{21}$$

and

$$p_t^s = \frac{\sum_{t-(ns-1)}^{i=t} \tilde{p}_i}{n_s} \tag{22}$$

The purpose of p_t^l is to measure long-term momentum and p_t^s to measure short-term momentum. The underlying philosophy of the trading system is that, whenever both short-term and long-term momenta become large in the positive or negative sense, then a position is taken in the direction opposite to the momentum; i.e., the momentum indicators are used as oscillators. The momentum-based trading rules are as follows. At the close of day t

1. if no position is held in the market, and
   - a. if $p_t^l < -b_t^l$ and $p_t^s < -b_t^s$, buy one contract, or
   - b. if $p_t^l > +b_t^l$ and $p_t^s > +b_t^s$, sell one contract;
   - c. otherwise, do nothing.

2. if a long position is held in the market, and
   - a. if either $p_t^l > +b_t^l$ or $p_t^s > +b_t^s$, close out the position, or
   - b. if $p_t^l > +b_t^l$ and $p_t^s > +b_t^s$, close out the position and sell one contract;
   - c. otherwise, do nothing.

3. if a short position is held in the market, and
   - a. if either $p_t^l < -b_t^l$ or $p_t^s < -b_t^s$, close out the position, or
   - b. if $p_t^l < -b_t^l$ and $p_t^s < -b_t^s$, close out the position and buy one contract;
   - c. otherwise, do nothing.

The trading system is applied to a sample of data beginning on day t for a number of days n_{out}, and its profitability is evaluated. The five adjustable parameters of the trading system are determined by first optimizing the profits generated by the system in a sample of data of size n_{in}

ending on day $t-1$.[8] The procedure is then re-applied with day t being replaced by day $t + n_{out}$. The procedure is followed until the trading system has been applied to the entire data set under consideration. At the end of each trading period, all trades are closed out. This insures that profits generated in different periods are independent of each other. One benefit of this methodology is that profitability of the trading system is assessed for a number of different, non-overlapping samples, each with its own optimized parameters. This should help in avoiding the chance selection of a profitable set of adjustable parameters. Another benefit is that the adjustable parameters are allowed to evolve in time, reflecting possible changes in market conditions.

Because of practical considerations, some of the parameters of the trading system have been assigned in an ad hoc manner. First, the size of each in-sample, n_{in}, used for optimization has been set to 250 days (approximately one year). The size of each out-of-sample, n_{out}, used for evaluation has been set to 66 days (approximately one quarter).

For the first subset of data, this results in 22 independent samples for evaluating profitability. These samples consist of points [316, 381], [382, 447], [448, 513], ..., [1,702, 1,767]. Points [1, 66] are used for initializing average momenta,[9] and points [66, 315] are the first set of points used for optimizing the adjustable parameters which are applied in the first out-of-sample [316, 381]. For the second subset this results in 24 independent samples for evaluating profitability. These samples are [1,768, 1,833], [1,834, 1,899], [1,900, 1,966], ..., [3,286, 3,352].

The calculations for performing optimizations are computationally intensive. To make the computations manageable, the search over values of the adjustable parameters which optimize the profits of the trading system have been restricted.[10] The adjustable parameters and the restrictions placed on their search values are listed in Table 3.

If, in a given search, several different sets of parameters yield the same optimum profits, then the set of parameters to be used for out-of-sample testing is selected by applying each of the following criteria, successively, until only one set of parameters remains: select the set with the smallest value of (1) n_d, (2) n_l, (3) n_s, (4) b_l, (5) b_s.

The trading system has been applied to the two subsets of data, and its profits, along with the profits resulting from a BAH strategy, are presented in Tables 3.2 and 3.2.[11] In addition, for the first subset the trading profits from the trend-following trading system of the study of EAO are also presented. Concerning the first subset it is apparent that the profits of the C2M trading system are greater than those of the trend-following system. However, it appears that the C2M trading system performs slightly worse than BAH. However, the C2M trading system holds positions in the market for only 788 of the 1,451 trading days during this period. A more appropriate way to contrast this trading system to BAH is to compare profits only during those periods when the system holds positions in the market. The results of this comparison are presented in Table 3.2. Note in the table that short trades, overall, are profitable in spite of the fact that the general trend of the market is up. This suggests that the C2M trading system may be effective in selecting

[8] A sufficient number of points, the maximum of n_d (see Table 3), from the beginning of the series are excluded from the analysis and used for obtaining initial values of average momenta.

[9] In addition, 1,000 data points have been prepended to the data set to calculate an initial value of the rescaled range (see "Persistent vs. Anti-Persistent Behavior and the Modified *R/S* Test").

[10] Approximately one day of computing time has been required to perform the optimizations on the two samples. Computations have been performed by a dedicated HP 735/125 workstation operating at approximately 55 Mflops.

[11] Profits are presented in both S&P 500 points and dollars. Note that prior to November 3, 1997, one S&P 500 point was valued at $500. On that day the value of one point became $250. The number of positions carried through November 3 was doubled, i.e., open interest doubled, to compensate for the reduction in value of the contract. All analyses in this study are done assuming that initially one contract is traded. On November 3 and subsequent days, the number of contracts traded is two. Thus, for purposes here, one S&P 500 point continues to be equivalent to $500.

TABLE 3. PARAMETERS TO BE OPTIMIZED AND RESTRICTIONS ON THEIR SEARCH VALUES.

Parameter	Restrictions
n_d (Eq. 16)	22, 33, 44, 55, 66
n_l (Eq. 21)	1, 2, 3, ... , 20
n_s (Eq. 22)	1, 2, 3, ... , 20
b_l (See trading rules.)	$0, 1 \times 2.88, 2 \times 2.88, 3 \times 2.88, \ldots 19 \times 2.88$
b_s (See trading rules,)	$0, 1 \times 2.88, 2 \times 2.88, 3 \times 2.88, \ldots 19 \times 2.88$

profitable trades. Unfortunately, the variability of the profits among trades makes the average profit per trade not statistically different from zero.

For the second subset of data, the C2M trading system appears to be inferior to BAH. Buy-and-hold is superior even when comparing profits only when the C2M system holds a position in the market (see Table 7.).[12] It is apparent from inspecting Figure 2 that it would be difficult for a trading system to outperform BAH during this period.

It can be argued that, for the first subset of data, the C2M system is as good as or perhaps even slightly better than BAH. This is true even when slippage and commissions are taken into account, for which a fair value would be about $50.[13] Even BAH will incur commissions since contracts will need to be rolled over at their expiration. For the second subset of data, the C2M system is clearly outperformed by BAH. In the next section the trading system is modified, improving profits in the second subset of data.

Modified Momentum-Based Trading System

As Murphy discusses (Murphy 1986, p. 276), an oscillator should be used only as a secondary indicator. Murphy points out that trend analysis is of primary importance. Consequently, it may be possible to improve the momentum-based trading system by including a

TABLE 4. TRADING PROFITS FOR THE FIRST SUBSET OF DATA

Contract	Profit	Profit/Trade	Max. Drawdown	No. of Trades
C2M	+127.30	+2.27	-34.05	56
	+$63,650	+$1137	-$17,025	
Trend (EAO)	-67.10	-1.46	-150.25	46
	-$33,550	-$730	-$75,125	
BAH	+63.45	-	-	-
	+$31,725			

Notes: Profits are given in both S&P 500 points and dollars. The calculation of maximum drawdown is based on closed trades.

[12] As in the case of the first subset, the average profit (loss) per trade resulting from short trades is not statistically different from zero.

[13] This value for transaction costs does not include the fact that some of the trades would, in practice, need to be rolled over to avoid trading in the expiration month. Thus, a more realistic estimate of transaction costs would be $60, since, on average, every trade actually corresponds to about 1.2 trades when rollovers are included.

TABLE 5. TRADING PROFITS FOR THE SECOND SUBSET OF DATA

Contract	Profit	Profit/Trade	Max. Drawdown	No. of Trades
C2M	-60.10	-.90	-350.40	67
	-$30,050	-$449	-$175,200	
BAH	+708.50	-	-	-
	+$354,250			

Notes: Profits are given in both S&P 500 points and dollars. The calculation of maximum drawdown is based on closed trades.

TABLE 6. TRADING PROFITS FOR THE FIRST SUBSET OF DATA

	Profits During Periods of			
	Short Trades	Long Trades	Total Profits	No. of Trading Days
C2M	+42.85 (31)	+84.45 (25)	+127.30	788
	+$21,425	+$42,225	+$63,650	
BAH	-42.85	+84.45	+41.60	788
	-$21,425	+$42,225	+$20,800	

Notes: Comparisons are made only during periods when the trading system holds a position in the market. Of particular interest is the comparison to BAH when the trading system is short. The values in parentheses are numbers of trades. The total number of possible trading days is 1,451.

TABLE 7. TRADING PROFITS FOR THE SECOND SUBSET OF DATA

	Profits During Periods of			
	Short Trades	Long Trades	Total Profits	No. of Trading Days
C2M	-104.60 (35)	+54.50 (32)	-60.10	864
	-$52,300	+$27,250	-$30,050	
BAH	+108.00	+67.80	+175.80	864
	+$52,300	+$27,250	+$74,550	

Notes: Comparisons are made only during periods when the trading system holds a position in the market. Of particular interest is the comparison to BAH when the trading system is short. The values in parentheses are numbers of trades. The total number of possible trading days is 1,583.

component that predicts the direction of the trend. The hedge statistic, I (Equation 15), is used for this purpose. Specifically, if $I > 0$, the trend is predicted to be up; if $I < 0$, the trend is predicted to be down; if $I = 0$, no direction is predicted for the trend. The hedge statistic is then used to filter trades; i.e., no long positions are initiated whenever $I \leq 0$, and no short positions whenever $I \geq 0$. All positions are exited as described in the previous section. In Tables 8, 9, 10, and 11, the modified trading system is compared to BAH. The modified trading system appears to perform better than the trading system without a trend component. For example, in the first subset of data, the profit per trade has increased from +2.27 S&P 500 points to +2.89. In the second subset it has increased from -.90 to +5.46. More important, the modified trading system performs, overall, at least as well as BAH. This is evidenced by the fact that in both subsets of data the modified trading system shows a net profit for short trades. This is especially noteworthy in the second subset, where the trend has been decidedly up. In both the first and second subsets, the overwhelming majority of the trades have been long. With hindsight this is expected since, in both subsets, the trend has been up. Another point is that the modified trading system does not hold a position in the market most of the time. In fact, for the first subset the system is trading for about 18 percent of the total possible trading days and garners 146 percent of the profits of BAH. For the second subset it trades about 20 percent of the time, showing profits of about 25 percent of BAH. In conclusion, the results suggest that the modified trading system is at least as good as BAH and perhaps better. This conclusion is discussed in further detail in the next section. A final point is whether the trading system could have been implemented in practice, yielding comparable trading profits. The answer is *yes*. Since this trading system is similar in some respects to that presented in the study of EAO, the arguments presented there apply here. The principal negative aspect of this trading system is that it may not manage risk at a level acceptable to most traders, as is evidenced in the drawdowns given in Tables 8 and 9.

Comments about Market Timing and Investment Performance

The modified C2M trading system has been presented as an alternative to BAH. The basic tenet is that performance can be improved by judiciously selecting when to buy or sell the market. Market timing strategies have been discussed extensively. See, for example, the two-part article of Merton and Henriksson (Merton 1981; Henriksson and Merton 1981). In part one (Merton 1981), Merton introduces the theoretical framework of a general market timing model and shows how timing strategies subsumed by the model are equivalent to certain option strategies. The model includes a broad base of trading strategies designed to outperform BAH by identifying when equities are under- or overvalued with respect to fixed income securities. Excluded from consideration, however, are timing strategies that utilize short-selling. In part two (Henriksson and Merton 1981), Henriksson and Merton develop statistical procedures for testing the performance of trading strategies within the context of their model. Although there are similarities between the modified C2M trading system and those timing strategies included in Merton's model, there is an important difference: the modified C2M system utilizes short selling to outperform BAH, rather than investing in fixed income securities. Thus, whether the modified C2M system outperforms BAH depends primarily on whether the short trades of the C2M system significantly improve its performance over BAH. This cannot be directly resolved using the statistical procedures of Henriksson and Merton, and indeed, resolving this unambiguously is beyond the scope of this paper. However, the statistical tests proposed by Henriksson and Merton suggest a method of partially resolving this.

During the time frame considered in this study, the modified C2M trading system has initiated only nine short trades, three within the first subset of data and six within the second. The three within the first subset are profitable, and three of six within the second subset are profitable. Three out of three profitable short trades may suggest profitability, but three out of six profitable trades seems to be no better than random chance. However, because of the extreme uptrend within the second subset it appears that random selection of entry and exit points for a short position would be more likely to result in a losing trade. Thus, three out of six profitable trades in the second subset is better than random chance. How much better is not straightforward to determine, because the appropriate null hypothesis for comparison is not obvious. Nonetheless, assume the null hypothesis to be that short trades are equally probable to be either profitable or unprofitable. Therefore, the number of profitable trades would be binormally distributed with $p = q = .5$, where p and q are the probabilities of a trade being profitable and unprofitable, respectively. If the number of short trades of both subsets are combined, there will be six profitable and three unprofitable. Under the null hypothesis the probability, P, of obtaining six or more profitable trades is

$$P = \sum_{n=6}^{9} \frac{9!}{n!(9-n)!}(.5)^n(.5)^{(9-n)} = \frac{130}{512} = .254 \quad . \tag{23}$$

Thus, there is a 25 percent chance of this occuring, assuming the null hypothesis. Given the overly conservative nature of the null hypothesis, this result provides some quantitative evidence that the modified C2M trading system outperforms BAH. To obtain a more appropriate null hypothesis, i.e., more accurate values of p and q, would require Monte Carlo simulations.

TABLE 8. PROFITS OF THE MODIFIED TRADING SYSTEM FOR THE FIRST SUBSET OF DATA

Contract	Profit	Profit/Trade	Max. Drawdown	No. of Trades
C2M	+92.60	+2.89	-41.25	26
	+$46,300	+$1,447	-$20,625	
BAH	+63.45	-	-	-
	+$31,725			

Notes: Profits are given in both S&P 500 points and dollars. The calculation of maximum drawdown is based on closed trades.

TABLE 9. PROFITS OF THE MODIFIED TRADING SYSTEM FOR THE SECOND SUBSET OF DATA

Contract	Profit	Profit/Trade	Max. Drawdown	No. of Trades
C2M	+174.80	+5.46	-67.05	32
	+$87,400	+$2,731	-$33,525	
BAH	+708.50	-	-	-
	+$354,250			

Notes: Profits are given in both S&P 500 points and dollars. The calculation of maximum drawdown is based on closed trades.

Table 10. Trading profits of the modified trading system for the first subset of data.

	Profits During Periods of			
	Short Trades	Long Trades	Total Profits	No. of Trading Days
C2M	+15.00 (3)	+77.60 (23)	+92.60	264
	+$7,500	+$38,800	+$46,300	
BAH	-15.00	+77.60	+62.60	264
	-$7,500	+$38,800	+$31,300	

Notes: Comparisons are made only during periods when the trading system holds a position in the market. Of particular interest is the comparison to BAH when the trading system is short. The values in parentheses are numbers of trades. The total number of possible trading days is 1,451.

TABLE 11. TRADING PROFITS OF THE MODIFIED TRADING SYSTEM FOR THE SECOND SUBSET OF DATA

	Profits During Periods of			
	Short Trades	Long Trades	Total Profits	No. of Trading Days
C2M	+.65 (6)	+174.15 (26)	+174.80	311
	+$325	+$87,075	+$87,400	
BAH	-.65	+174.15	+173.5	311
	-$325	+$87,075	+$86,750	

Notes: Comparisons are made only during periods when the trading system holds a position in the market. Of particular interest is the comparison to BAH when the trading system is short. The values in parentheses are numbers of trades. The total number of possible trading days is 1,583.

Discussion and Conclusions

Since the mid 1980s the S&P 500 stock and futures indices have increased in value appreciably, with some remarkable gains occurring during the '90s. The rate of return on the futures index has been approximately 10 percent compounded annually. Even including the commissions associated with rolling over contracts, many individuals would be content with such gains, which could have been realized by adopting a buy-and-hold strategy. If the S&P 500 index should sink into a protracted bear market, buy-and-hold may not be the strategy of choice, but rather some type of trading system may be. In this study a trading system has been proposed which seems to perform, at least, comparably to buy-and-hold. Furthermore, results of this study suggest that the system is profitable with short as well as long trades, presumably making the trading system appropriate for bear markets.

The trading system is based on the fact that the S&P 500 futures index exhibits anti-persistent behavior. Qualitatively this implies that large price moves in a given direction tend to be followed by price moves in the opposite direction. This is not a common trait of all futures markets. In fact, as shown in this study, the Japanese *yen* exhibits persistent behavior, implying that large price moves in a given direction tend to be followed by price moves in the same direction. Because of the anti-persistent behavior of the S&P 500 futures index, the trading system has been designed to buy on downturns and sell on upturns. Rules for entering or exiting positions are based on the values of short- and long-term momenta, formulae which are derived from Langevin equations. The trading system has a number of adjustable parameters, whose values are obtained by optimizing trading profits during a 250-trading-day period (approximately one year) prior to each 66-day period (approximately one quarter) in which the system has been applied. In order to improve the profitability, the trading system is modified to include a component for predicting the direction of the trend. The direction of the trend is predicted from the net open interest of the commercials (hedgers). Specifically, if commercials are net long, the trend is predicted up, and if commercials are net short, the trend is predicted down. Profits generated by the trading system are considerably improved by this modification, making the trading system, at least, as good as buy-and-hold. A corollary to this result is that this method of predicting the trend may be inconsistent with the idea of a positive risk premium. If one believes that hedgers sell their risk to speculators, and therefore speculators should be rewarded for assuming that risk, then the direction of the trend, on average, should be opposite to the net position of the hedgers.[14]

Although the results of this study are suggestive, they are not statistically rigorous. The obvious problem is the limited amount of data, making it impossible to replicate the study under identical market conditions. One way of circumventing this problem is to construct a model of the data that takes into account anti-persistence and dependencies on the net position of hedgers. In addition, the model should include effects of conditional heterogeneity, which is well known to be present in long series of financial data. The model could then be used to simulate market data. The trading system could be applied to the simulations, and statistically rigorous comparisons could be made to buy-and-hold. This is left for future investigation.

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[14] In an unpublished study it has been shown that, in the case of the Japanese *yen* and corn futures markets the direction of the trend is better predicted as opposite to the net position of hedgers. This seems to be more consistent with the idea of a positive risk premium.

The commitment of traders report. The Commodity Futures Trading Commission. The most recent, as well as previous editions of this report, are available at *http://www.cftc.gov/dea/cot.html*.