Physics 201 Lab: Standing Waves on a String

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Traveling Wave Fundamentals

A wave traveling in the positive x-direction can be described by the equation

$$y = f(x - vt) \tag{1}$$

The variable y is the wave disturbance at the coordinate x and at the instant t. v is the *phase velocity* of this traveling wave, and f is an arbitrary function that specifies the wave *profile*. The profile is simply the curve $f(x)$ representing the initial wave shape. Equation 1 says the wave shape is unaltered as the wave propagates with velocity v.

A particularly important class of traveling waves are *repetitive* waves. Such waves are generated by a periodic source, and have the property that the disturbance repeats both in space and time. Specifically, the disturbance at any fixed point will recur after a time T has elapsed: T is the *period* of the wave, and its reciprocal $1/T$ is the wave *frequency* (one complete vibration executes in exactly one period). Similarly, the disturbance at any instant repeats over a spatial interval of length λ: λ is called the *wavelength* of the wave. A general repetitive traveling wave can be constructed by adding together [with appropriate amplitudes] sine (or cosine) waves, each of which has the structure of Equation 1. These are *harmonic waves*: in terms of their wavelength and frequency, harmonic waves are described by the mathematical form

$$y = y_m \sin\left(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T} + \phi\right) \tag{2}$$

y_m is the amplitude (maximum value of the disturbance y) of this harmonic wave, and ϕ is its phase. Using the periodicity of the sine function ($\sin(\theta + 2\pi) = \sin\theta$), we verify that $y(x + \lambda, t) = y(x, t)$ for any time t, and $y(x, t + T) = y(x, t)$ at any position x. For our purposes, it will suffice to take $\phi = 0$.

Equation 2 can be re-written in the form

$$y = y_m \sin\left(\frac{2\pi}{\lambda}[x - \lambda f t] + \phi\right) \tag{3}$$

Here we have inserted the wave frequency $f = 1/T$ (not to be confused with the wave profile $f(x)$ introduced earlier). Comparison of Equation 3 with Equation 1 implies that the phase velocity is given by

$$v = \lambda f = \frac{\lambda}{T} \tag{4}$$

Standing Waves on a String

The phase velocity v of a wave traveling along a vibrating string can be found from the tension in the string τ and the [linear] mass density of the string μ as

$$v = \sqrt{\frac{\tau}{\mu}} \tag{5}$$

Equations 4 and 5 together show that for a string under tension τ, the wavelength λ of waves on the string must be

$$\lambda = \frac{v}{f} = \frac{1}{f}\sqrt{\frac{\tau}{\mu}} \tag{6}$$

If a sinusoidal source (the driver) is forcing the string at one end, and the other end of the string is fixed, the wave traveling down the string will be reflected at the fixed end. The reflected wave interferes with the incoming wave, producing a *standing wave* along the string. For such a wave, each string element vibrates at the source frequency with an amplitude that depends on its position; in particular, the standing wave nodes occur at fixed locations along the string. The nodal separation is one-half of a wavelength, and can be varied by adjusting the tension in the string. Unusually large vibration amplitudes result if a node coincides with the location of the driver (at one end of the string): this disproportionately large response to the driver stimulus is termed *resonance*. At resonance, the length L of the string must be equal to an integral number of half-wavelengths, as shown in Figure 1.

$$L = n\frac{\lambda}{2} \quad n = 1, 2, 3, \ldots \tag{7}$$

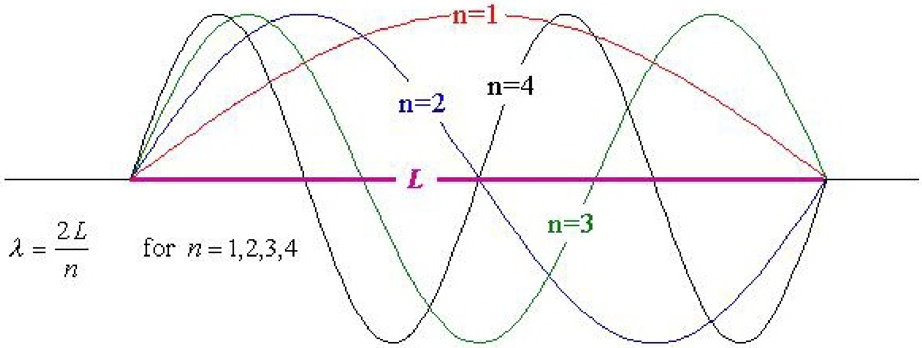

FIG. 1: Conditions for resonance on a string with a node at both ends.

n is the *mode number*, and the associated wavelength $\lambda = 2L/n$ is called the *modal wavelength*. Since the frequency f is set by the driver (and unchanging), each mode must have a different velocity v, corresponding to a different string tension τ.

Procedure

In today's lab, you will investigate the four longest-wavelength modes of a string vibrating at a fixed frequency by varying the tension required to create each mode. The tension is controlled by adjusting the mass hung from one end of the string. The experimental setup is shown in Figure 2. For this arrangement, the tension is simply the weight of the hanging mass, $\tau = mg$, so Equation 6 becomes

$$\lambda = \frac{1}{f}\sqrt{\frac{g}{\mu}}\sqrt{m} \tag{8}$$

A regression equation based on the form of Equation 8 is obtained using mass and wavelength data derived from the experiment. The consistency of the theory underlying Equation 8 is assessed by comparing the slope of the regression line to its theoretical value. The particulars of the experiment are as follows:

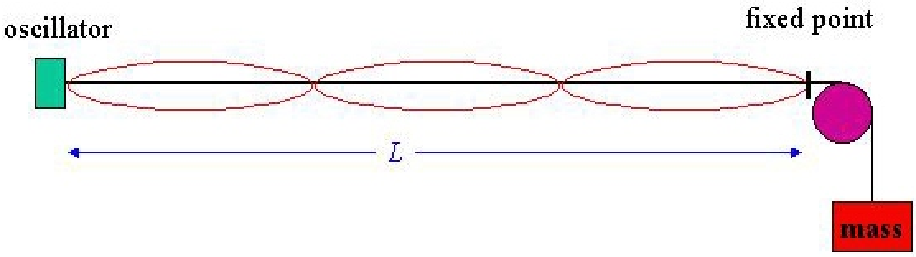

FIG. 2: Experimental setup for measuring the modal wavelengths of a vibrating string.

1. Add enough mass to the mass hanger so that the string supports a standing wave with just one node in the interior of the string. Adjust the mass on the hanger until resonance is achieved (large-amplitude vibration) for this, the $n = 1$ or *fundamental* mode. At resonance, the interior node should be as close to the driver-end of the string as your experimental technique will allow.

2. Measure the distance L from the node at the pulley to the interior node on the string. Record the result to the accuracy of your measurement (at least 3 digits). Also record the mass m for this mode, using the triple-beam balance. Be sure to include the mass of the hanger.

3. Repeat the preceding steps for $n = 2, 3, 4$, n being the number of nodes in the interior of the string. In each case, carefully adjust the hanging mass m to produce resonance, then measure m and the distance L from the node at the pulley to the interior node closest to the driver.

4. Using Equation 7, calculate the wavelength λ of the standing wave for each of the [four] cases studied. Note: since the distance between consecutive nodes is $\lambda/2$, the measured length L contains 1, 2, 3, and 4 half-wavelengths, respectively.

5. In Excel, plot a graph of λ vs $\sqrt{m}$. λ is considered the dependent variable (cf. Equation 8), and is plotted along the vertical (ordinate) axis. Perform a linear regression of the data, forcing the intercept to be zero. Display the equation and the correlation coefficient for the 'best fit' line.

6. Using Excel's LINEST function[1], find the slope (M) of your graph and the standard error in the slope (σ). [Since we forced the intercept to zero, the 3rd LINEST argument must be entered as 'FALSE'.] Report this experimental value of the slope in the form $M \pm \sigma$.

7. For comparison, calculate the theoretical value of the slope implied by Equation 8. The frequency of vibration is $f = 60$ Hz, and the mass density of the string is $\mu = 2.98 \times 10^{-4}$ kg/m. Take $g = 9.80$ m/s^2 for the acceleration due to gravity. Does the theoretical prediction fall within the experimental range?

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[1] Since LINEST is an *array function* (a single function call generates multiple outputs), the function call must be made in a special way: Select the cell containing the LINEST entry. Starting with the formula cell, highlight a block of cells (2 columns by 3 rows will suffice) to receive the output values, then press F2 (fn+F2 on Mac). Finally, press Ctrl+Shift+Enter. More details on using LINEST are available in Excel Help.