# Physics 201 Lab: Centripetal Force in Uniform Circular Motion

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## Theoretical Discussion

An object moving in a circular path at constant speed is said to be executing *uniform circular motion.* The basic kinematic variables that describe an object in uniform circular motion can be taken to be the radius $R$ of the orbit and the period $T$ of the orbital motion (the time required to complete one revolution). Closely related to the period is the frequency $f$ of the orbital motion. Frequency refers to the number of revolutions executed per unit time; since one revolution takes exactly one period, it follows that

$$f = \frac{1}{T} \tag{1}$$

The speed of a circulating object is often described in terms of its *angular velocity* $\omega$, which is the angle swept out by the object per unit time as it moves along its trajectory: $\omega = \Delta\theta/\Delta t$. If angle is swept out at a constant rate – as in uniform circular motion – then a full circle ($2\pi$ radians) is swept out in exactly one period, so

$$\omega = \frac{2\pi}{T} = 2\pi f \tag{2}$$

Also, with the help of Figure 1, we see that the arc length $\Delta s$ traversed by an object moving in a circular path of radius $R$ is related to the angle subtended at the center of the circle $\Delta\theta$ by the simple rule

$$\Delta s = R\Delta\theta$$

so long as $\Delta\theta$ is measured in radians.[1]

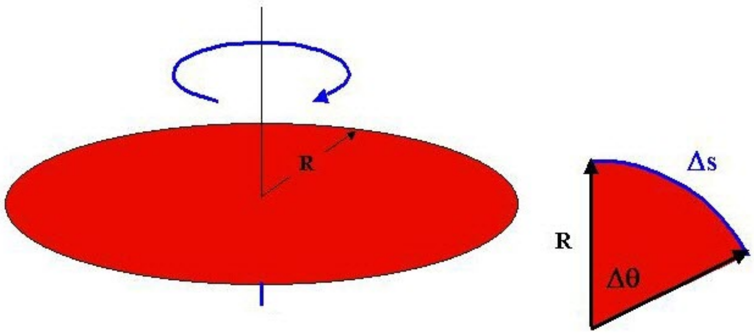


FIG. 1: Geometry describing rotational kinematic variables.

It follows that the [linear] velocity $v$ and the angular velocity $\omega$ are related as

$$v = \frac{\Delta s}{\Delta t} = R\frac{\Delta\theta}{\Delta t} = R\omega \tag{3}$$

The direction of the velocity vector $\vec{v}$ is, of course, tangent to the circular path.

In the absence of forces, an object will travel at a constant speed in a straight line forever (Newton's 1st Law). The *only* way that an object can execute uniform circular motion is for a force to act on it in such a way that its speed never changes, but the direction of its velocity does. This is the *centripetal force*. Consider an object of mass $m$ which has a velocity of $\vec{v}(t_0)$ at time $t_0$, shown in Figure 2. At some later time $t_1 = t_0 + \Delta t$, the velocity is $\vec{v}(t_1)$. The magnitudes of $\vec{v}(t_0)$ and $\vec{v}(t_1)$ are the same, but their directions are different. By definition, the average force $\vec{F}_{avg}$ that changes $\vec{v}(t_0)$ into $\vec{v}(t_1)$ is given by

$$\vec{F}_{avg} = m\frac{\Delta \vec{v}}{\Delta t} = m\frac{\vec{v}(t_1) - \vec{v}(t_0)}{t_1 - t_0}$$

The three vectors $\vec{v}(t_0)$, $\vec{v}(t_1)$ and $\frac{(t_1 - t_0)}{m}\vec{F}_{avg}$ form the triangle shown in Figure 2. As the time interval $\Delta t$ becomes progressively smaller (Figures 3A and 3B), the average force becomes simply $\vec{F}$ (the centripetal force), and is directed towards the center of the circle. The magnitude of this force is

$$F = m\frac{v^2}{R} = m\omega^2 R$$

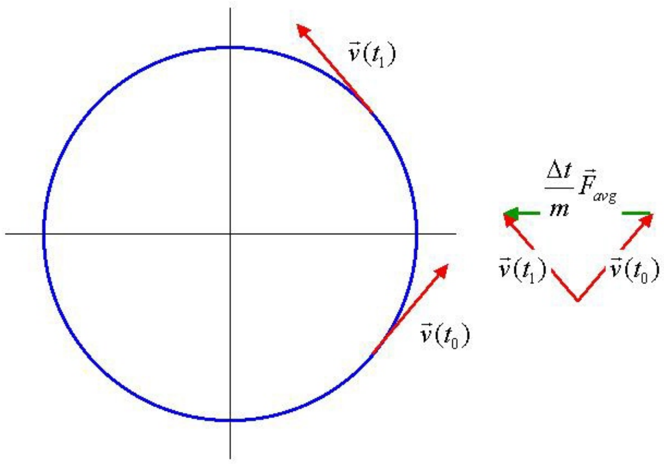


FIG. 2: Change in velocity over a finite time interval.

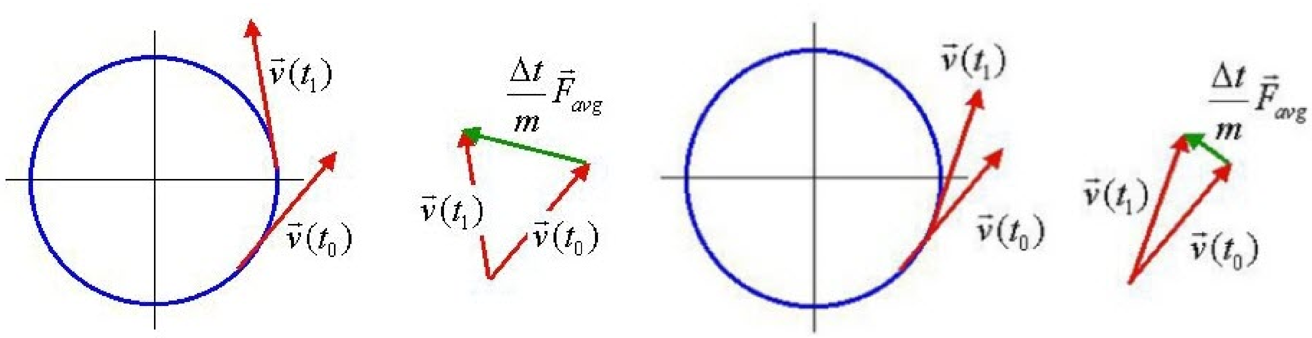


Figure 3A

Figure 3B

FIG. 3: Direction of centripetal force in the limit as $\Delta t \to 0$.

## Experimental Procedure

In today's lab, you will calculate the magnitude of the centripetal force that is required to provide the centripetal acceleration of an object in uniform circular motion, and compare your result with a 'direct' measurement of this force using an equilibrium technique. The experimental setup is shown in Figure 4. Uniform circular motion is achieved by spinning the rotor arm at a constant rate such that the radius of the circular orbit of the suspended mass remains constant. Procedural details are as follows:

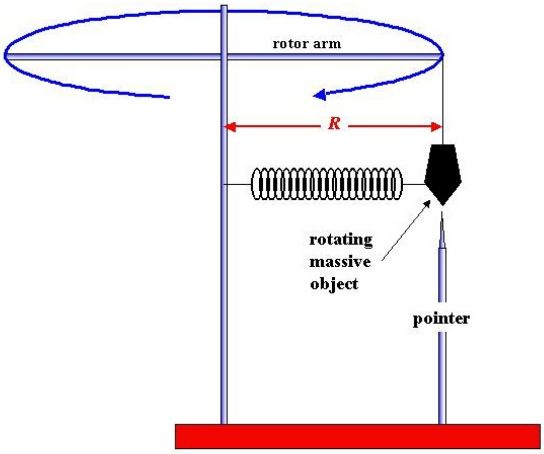


FIG. 4: Experimental setup for the measurement of centripetal force

Note: Align the apparatus so that with the mass hanging vertically, the [stretched] spring is horizontal. (Wrap or unwrap the supporting thread as necessary to achieve this.)

1. The radius of the orbit $R$ is equal to the distance from the axis of rotation to the pointer when the mass is disconnected from the spring and therefore, free to hang straight down. Adjust the pointer so that it is vertically aligned with the pointed end of the mass when the spring is disconnected. Measure and record the orbit radius $R$.

2. Measure the mass $m$ without the spring attached.

3. After re-connecting the spring, spin the rotor arm – at as constant a rate as possible – so that the mass is vertically aligned with the pointer. It is important that the mass is hanging straight up and down, because then the string from which the mass is suspended exerts no component of force in the radial direction; the force keeping the mass in uniform circular motion is therefore supplied completely by the spring. When this condition is achieved, time (using a stopwatch) how long it takes for the mass to execute [at least] 20 rotations. Both team members should perform this timing measurement. Record the individual measurements as $t_1$, and $t_2$, and find their average $t_{avg} = \frac{1}{2}(t_1 + t_2)$.

4. Calculate and report the period $T$, the frequency $f$, the angular velocity $\omega$ and the linear speed $v$ of the orbiting mass, using Equations 1–3. From these results deduce the magnitude of centripetal force $F_{cent}$ that accounts for the centripetal acceleration of the orbiting mass:

$$F_{cent} = m\omega^2 R$$

5. In order to check this prediction, you must make an independent measurement of the force exerted by the spring when the mass is vertically suspended over the pointer.

The magnitude of the force $F_{spring}$ exerted by a spring depends on the nature of the spring and the amount by which it is stretched. To measure the force exerted by the spring during the course of the experiment, you will determine the weight force $W$ required to extend the spring until the mass is vertically suspended over the pointer when the rotor arm is *not* turning. As shown in Figure 5, if we attach a mass hanger to the (non-)rotating mass and drape the hanger over a pulley, then suspend some weights from the mass hanger until the non-rotating object is vertically aligned with the pointer, the weight force $W$ produced by gravity 'pulling' on the suspended mass $M$ is equal in magnitude to the force exerted by the spring when the object is rotating:

$$F_{spring} = W = Mg$$

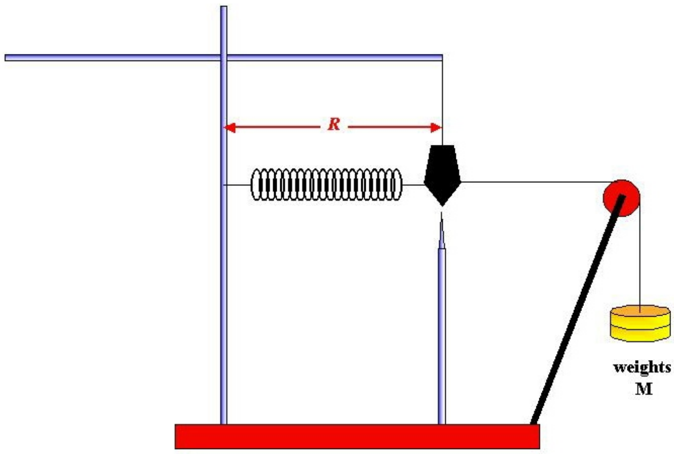


FIG. 5: Determining the centripetal force by imposing static equilibrium with gravitational forces.

Report your measured value for the mass $M$ of the weight (including the hanger) required to counter-balance the spring force and position the object over the pointer; use this value to calculate $F_{spring}$.

6. Finally, calculate and report the discrepancy [difference] between $F_{cent}$ and $F_{spring}$; express your result as a percentage of $F_{spring}$ (presumably the better known of the two).

**Hint:** It would be wise to convert all your measurements to SI units before making any calculations.

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[1] In fact, this equation defines what we mean by the radian measure of an angle as the ratio of arc length to circle radius.