Physics 201 Lab 9: The Conservation of Linear Momentum

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Theoretical Discussion

The principle of conservation of momentum states that

The total momentum of an isolated system is constant.

While both the momentum and energy of an isolated system are always conserved, it may be that some of the energy in a collision is converted to internal energy of one or more of the colliding objects. It may appear that energy is not conserved, but this is not the case. It is merely that this "lost" energy no longer has the form either of kinetic or potential energy. This internal energy is hard to keep track of and may not be directly recoverable as kinetic or potential energy because it has dissipated throughout the particles making up the system. Nonetheless, even in such collisions both the total momentum and the total energy are still conserved. Collisions in which the total momentum is obviously conserved but some part of the kinetic or potential energy is converted to internal energy of the objects making up the system are known as *inelastic* collisions. Collisions in which none of the energy is converted to internal energy, so that the conservation of energy is also obvious, are known as *elastic* collisions. In today's lab we will study elastic collisions in one dimension.

Binary elastic collision in one dimension

One particular type of elastic collision is the "rear-end" collision, in which a moving object of mass m_1 and initial velocity $\vec{v}_1$ rams another object of mass m_2 which is at rest. For a 'head on' collision of this type, the colliding objects both move along the initial direction of motion, so the collision is one-dimensional. In this case, conservation of momentum and energy become (using lower and upper case for the velocities before and after collision, respectively)

$$m_1 v_1 = m_1 V_1 + m_2 V_2 \tag{1}$$

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2 \tag{2}$$

In the aftermath of the collision, there are two unknown velocities (V_1 and V_2), and two equations to determine them. Thus, conservation principles alone are enough to specify the outcome.

Equations 1 and 2 must be solved simultaneously. The process is straightforward, but tedious. In terms of the colliding masses and v_1, the initial velocity of object 1, we find

$$V_1 = \frac{m_1 - m_2}{m_1 + m_2} v_1 \tag{3}$$

$$V_2 = \frac{2m_1}{m_1 + m_2} v_1 \tag{4}$$

If their masses are equal, the two colliding objects simply trade velocities ($V_1 = 0$ and $V_2 = v_1$), whereupon it is evident that both momentum and energy are conserved. Also, we see that $V_2 > 0$, so that the struck object always moves in the same direction as the striking object's initial velocity, as expected.

Subtracting Equations 3 and 4 furnishes another interesting (though not independent) relation:

$$V_2 - V_1 = v_1 \tag{5}$$

The quantity on the left is just the *separation speed* of the two objects after collision, while that on the right is their *closing speed* before collision; that these two are equal is a hallmark of elastic collisions. At the other extreme of a perfectly inelastic collision, the separation speed is zero. More generally, the ratio of separation speed to closing speed in one-dimensional collisions is the *coefficient of restitution*, denoted by e, a dimensionless quantity that measures the degree of elasticity and varies between 0 and 1. Some further observations of note also follow readily by inspection if we distinguish the two cases $m_1 > m_2$ and $m_1 < m_2$:

1. Case: $m_1 > m_2$ (striking object has larger mass)
   - $V_1 > 0$ always (striking object continues to move in the "forward" direction), and approaches v_1 in the limit $m_2 \to 0$ (struck object mass is negligible). Also, a separation speed greater than zero (cf. Equation 5) guarrantees that the striking object never overtakes the struck object after the collision takes place.
   - V_2 approaches $2v_1$ in the limit $m_2 \to 0$ (struck object mass is negligible).
2. Case: $m_1 < m_2$ (struck object has larger mass)
   - $V_1 < 0$ always (striking object reverses direction), and approaches $-v_1$ in the limit $m_1 \to 0$ (striking object mass is negligible).
   - $V_2 < v_1$ always, and $V_2 \to 0$ in the limit $m_1 \to 0$ (striking object mass is negligible).

In today's experiment we will investigate both unequal-mass cases.

Experimental Procedure

In this experiment, you will crash one air cart into another, initially at rest. The experiment is illustrated in Figure 1. You will measure the time intervals during which the 'sails' of the air carts block the infrared beams of the photogates, which will enable you to calculate their speeds both before and after collision. To convert these times into speeds, you will need to measure the width of the sails. The speeds are then just the sail widths divided by the time intervals.

The particulars of the experiment are as follows:

1. Using the vernier calipers, measure the width of the sail carried by each air cart. The calipers affords 4-digit accuracy, so your recorded values should have 4 significant figures.
2. Make sure the air track is horizontal (a crossbar support near one end is provided for that purpose). Place one of the carts on the track near the middle of the 'active' section, and turn on the air flow. Make adjustments as necessary to level the track: when the track is level, the cart shows no tendency to 'drift' preferentially one way or the other.
3. Turn on the photogate timer and place it in S_1 mode.[1]
4. Crash air cart 1 into air cart 2. Cart 2 will be initially at rest at some point directly in between the two photogates. Cart 1 will pass through photogate 1 prior to the collision, will strike cart 2, and then will either move forward or backward depending on the relative masses of the two carts. After collision, both carts will move through one or the other photogate. Because the timer will read back the time intervals in the order in which they were measured, you must note the sequence in which the air carts move through the gates in order to correlate the times with the carts. After you have taken your measurements, push the *stop* button. The timer will then sequentially display the measured intervals in the order in which they were recorded. You should have recorded three time intervals. Use your timing and sail-width measurements to calculate v_1, V_1, and V_2 for this collision.

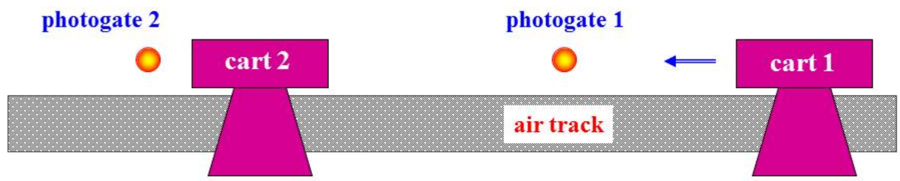

FIG. 1: Experimental setup for the measurement of momentum conservation

5. Repeat the previous step for two more collision speeds. **In this and the next step, be sure that all collisions occur between the same pair of cart 'bumpers'.**

6. Interchange the two carts and repeat the preceding step for two additional collision speeds, in this way gathering data for both cases $m_1 > m_2$ and $m_1 < m_2$. [Note that the label 1 in our equations *always* refers to the incoming cart, and 2 to the (initially stationary) target cart.] You should now have data for five distinct collisions. Use different collision speeds for all five trials, so as to obtain as much variation in this parameter as the equipment and your technique will allow.

7. To investigate the elasticity of the collisions, make a graph (in Excel) of separation speed vs. closing speed (defined in Equation 5), using the data from all five trials. Perform a linear regression of the data, forcing the intercept to zero. Use Excel's LINEST function[2] to find the slope of your graph (e) and the standard error in the slope (σ). [Since we forced the intercept to zero, the 3rd LINEST argument must be entered as 'FALSE'.] The slope is your experimental determination of the restitution coefficient for the carts used in the experiment. Report your results in the form $e \pm \sigma$.

8. If you haven't yet measured the masses of the two carts, do so now and record the results to 4-digit accuracy. To check momentum conservation, subtract the left and right-hand sides of Equation 1 to find their difference. Report the difference as a percentage of the available momentum, i.e., the momentum carried by the incoming cart. Do this separately for all five trials.

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[1] The function mode is altered by repeatedly pushing the *function* button; an LED indicates which function is activated. A second LED indicator identifies the units in which the result will be output on the display panel. Press the *clear* button to clear any previous data from the timer's memory buffers.

[2] Since LINEST is an *array function* (a single function call generates multiple outputs), the function call must be made in a special way: Select the cell containing the LINEST entry. Starting with the formula cell, highlight a block of cells (2 columns by 3 rows will suffice) to receive the output values, then press F2 (fn+F2 on Mac). Finally, press Ctrl+Shift+Enter. More details on using LINEST are available in Excel Help.