Physics 201 Lab 1: Methods for Experimental Physics

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Errors and Precision

There are two different sources of error in any measurement. The first is called *precision*. Precision refers to how fine your measurement scale is. A general rule of thumb is that you can measure to within 1/2 of the finest division on your measuring device. Thus, for example, if your measuring stick has divisions of 1 mm, your precision is ±0.5 mm. You should not apply this rule mindlessly, however; Even if your measuring stick has 1 inch divisions, you can probably measure things more precisely than ±0.5 in.

The second is called *measurement error*. It refers to your intrinsic ability to make the measurement with the tools at hand. For instance, even if your ruler were demarked in microns (10^{-6} m), you couldn't measure something to that precision if you had to use your naked eye to do it—you simply couldn't see that well. There are many possible causes for measurement error, and there are no simple rules to tell you how big it is. Usually, you simply have to use your own judgement to estimate it.

The result of any measurement should be accompanied by your estimate of the uncertainty (combined precision and measurement error) in that measurement. The measured value should be reported to the number of significant digits consistent with that uncertainty (generally, it is understood that there is some uncertainty in the last recorded digit).

Systematic and Random Errors

Measurement errors are further classified into systematic errors and random errors. *Systematic errors* are errors that deviate from the true value by a predictable amount, such as improperly calibrated equipment. *Random errors* are unpredictable and often unavoidable, like errors caused by fluctuations in temperature, pressure etc. Systematic errors can be corrected if the cause is known; random errors are hard to identify and correct, but lend themselves to statistical analysis. For a truly random process, the values obtained in repeated measurements of the same quantity can be shown to follow the *normal distribution* ('bell' curve) shown in Figure 1.

This distribution is characterized by just two parameters. The *mean* μ is the value obtained most often in a measurement of the quantity under study, and locates the peak of the distribution. The *standard deviation* σ indicates the spread of the distribution, i.e., how much the measurements deviate from the mean. The mean is the single number that best represents the quantity being measured; the standard deviation allows you to predict the liklihood that a single measurement will reproduce the mean value, or something close to it. For example, according to Figure 1, 68% of all measurements fall within the range $[\mu - \sigma, \mu + \sigma]$; this is known as a 68% *confidence interval*.

The normal distribution is a hypothetical construct that refers to an infinite number of measurements. In reality, only a finite number of measurements, say N, are available: we will denote the measured values as $x_1, x_2, x_3, ..., x_N$. A straightforward average of those measurements approximates the true mean with an accuracy that should improve with larger values of N:

$$x_{avg} = \frac{x_1 + x_2 + x_3 + ... + x_N}{N}$$

In much the same way, only an approximation to σ is afforded by this finite data set. To obtain the *rms (root-mean-square) deviation* approximation to σ we average the squares of the differences between each individual measurement and the measurement average, then take the square root of the result:

$$\sigma_{rms} = \sqrt{\frac{(x_1 - x_{avg})^2 + (x_2 - x_{avg})^2 + ... + (x_N - x_{avg})^2}{N}}$$

As it happens, a better approximation for small data sets obtains by replacing N with $N-1$ in this formula. σ_{rms} is often used to find the *standard deviation of the mean* σ_m:

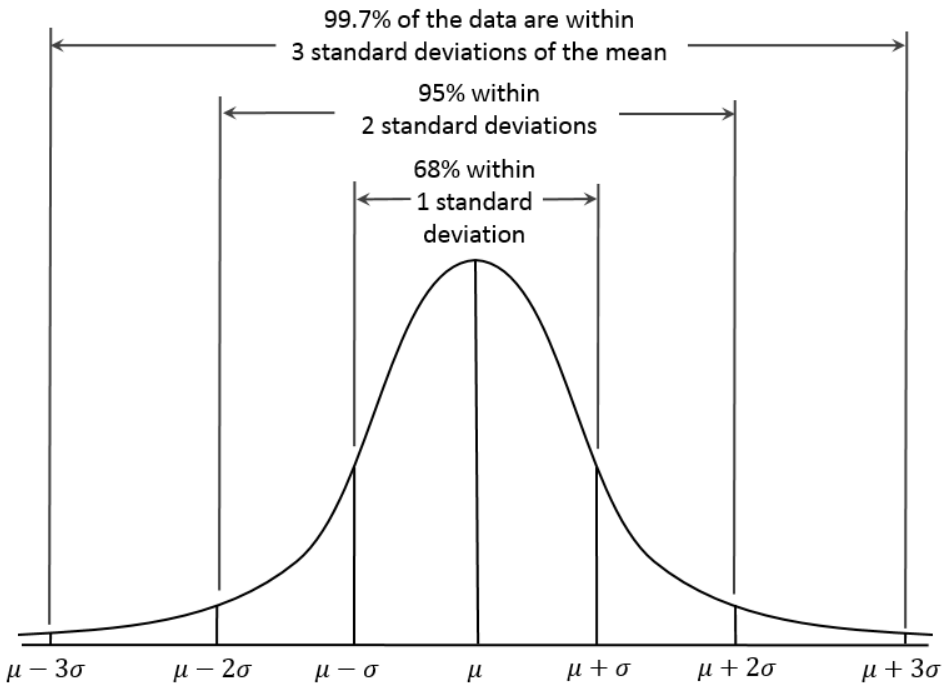

FIG. 1: The normal distribution for random errors

$$\sigma_m = \frac{\sigma_{rms}}{\sqrt{N}}$$

Armed with σ_m, we can report confidence intervals for the average value calculated from our measurements as $x_{avg} \pm \sigma_m$(68% confidence interval), or $x_{avg} \pm 2\sigma_m$(95% confidence interval), etc.

Combining Measured Values

When measured values are used in calculations, the result is uncertain by an amount that reflects the error in the measurements. It is nonsensical to report a result to an accuracy greater than that of the inputs used to compute it, but how much uncertainty should we attach to a calculated value? The rules for manipulating significant digits serve as a useful guide in this regard, but the full story is typically far more involved, as the following example illustrates.

Suppose that you must measure a reasonably large area. The area is divided up into a square grid, with some remainder in the length and width. If you had a big enough measuring stick, you could measure it directly. But a simpler procedure, and one that is possibly more accurate as well, would be to count the number of whole squares along the length and width, and add in the appropriate remainders.

For instance, suppose there are 8-$\frac{1}{4}$ squares along the length, and 4-$\frac{1}{2}$ squares along the width. Each square is 0.5 m to a side, so the length is $l = (8.25)(0.5) = 4.125$ m. Similarly, the width is found from $w = (4.5)(0.5) = 2.25$ m. What about the uncertainty? Lets say you have measured the squares to an accuracy of 2 mm. Since your length contains 8-$\frac{1}{4}$ squares, the uncertainty in the length is $\Delta l = (8.25)(0.002) = 0.0165 = 0.017$ m. We have left off the final digit (5) because the total uncertainty for 8-$\frac{1}{4}$ squares cannot be specified more precisely than the uncertainty for one square. The uncertainty in the width is $\Delta w = (4.5)(0.002) = 0.009$ m. Thus you should report the dimensions of the length and width as $l = 4.125 \pm 0.017$ m and $w = 2.250 \pm 0.009$ m.

What about the area? The area itself is easy: $A = (4.125)(2.25) = 9.2813$ m^2. What about the uncertainty in the area? Conceivably, you could have been off in your determination of the area by as much as $\Delta A =$

$l\Delta w + w\Delta l = (4.125)(0.009) + (2.25)(0.017) = 0.075375$ m^2. We could round this either to $\Delta A = 0.08$ m^2, or to $\Delta A = 0.07$ m^2. In the first case, we might overstate our uncertainty; in the latter we might understate it. Therefore, we choose to include the second figure to be more precise about our uncertainty, so that we take $\Delta A = 0.075$ m^2. It doesn't make any sense to be more precise than this. Since you are uncertain in the area by at least 0.075 m^2, it makes no sense to report the last decimal (3) of your area, because your uncertainty is much larger than this. In general, you should report your result to the same precision as you report your uncertainty. Therefore, you would report your area as

$$A = 9.281 \pm 0.075 \text{ m}$$

This involves a lot more thinking than simple rules about how many digits to keep, but it is a more precise rendering of your state of knowledge of the area, and that is what other scientists reading your results need to know.

Graphing Data

When graphing data, there are some simple, but important rules and conventions that you should follow.

Conventions: The horizontal axis (sometimes called the"x-axis") is known as the *abscissa*. The vertical axis (sometimes called the "y-axis") is known as the *ordinate*. Sometimes we might wish to plot a variable called x on the vertical axis or a variable called y on the horizontal axis, so it is best to use the terms abscissa/ordinate or horizontal/vertical to avoid any confusion.

The instructions "Plot the mass vs. the frequency" and "Plot the mass as a function of frequency" both mean the same thing: Plot the mass on the ordinate (vertical axis) and the corresponding values of the frequency on the abscissa (horizontal axis).

Rules: Always label the graph axes so that anyone viewing your graph can readily comprehend what you have plotted. Your labels should include the units associated with the quantities plotted on each axis. Your graph should also include some kind of title that describes the relationship between the quantities plotted on the graph. In addition, you must properly scale your axes so that the relationship your graph explores is apparent to the viewer.

Scaling your graph: Adjust the scale and starting point of your axes so as to use as much of the page as possible. If all of your data is scrunched up into a tiny corner of your graph, you will not be able to see significant features of the data; the way one variable depends on the other, for instance.

Properly scaling your data is easy. Note that it is not necessary to use the same scale for both axes. Neither is it necessary for both scales to have the same origin, nor is it necessary for either scale to include the zero point. For each axis, you should:

- Determine the lowest and highest values for your data.
- Choose the lower and upper limits of your scale so that the lower limit is smaller than the lowest value of your data and the upper limit is larger than the highest value. You should also choose the limits so that the difference between them is an easy number to divide up.

As an example, consider Table I, which gives the yearly percentage change in population of a group of wallabees (on a wallabee farm) as a function of fat grams per liter in the wallabee feed they are given.

In order to determine the functional relationship between population change and dietary fat, we might plot the population change vs. feed fat content. The data in Table I suggest we scale the abscissa axis so that it goes between 500 and 900, and the ordinate axis so that it runs between -3.5 and 5.5. The resulting graph is shown in Figure 2.

Finding the equation describing linear data: If the data you plot on your graph seem to follow a straight line, it may be that a linear relationship exists between the variables. If we call the ordinate variable q and the abscissa variable t, then the equation of the line relating q and t is

$$q = at + b$$

fat content grams/liter	population change %
520	−3.2
590	−1.0
680	1.1
770	2.7
850	5.2

TABLE I: Effects of dietary fat on wallabee population

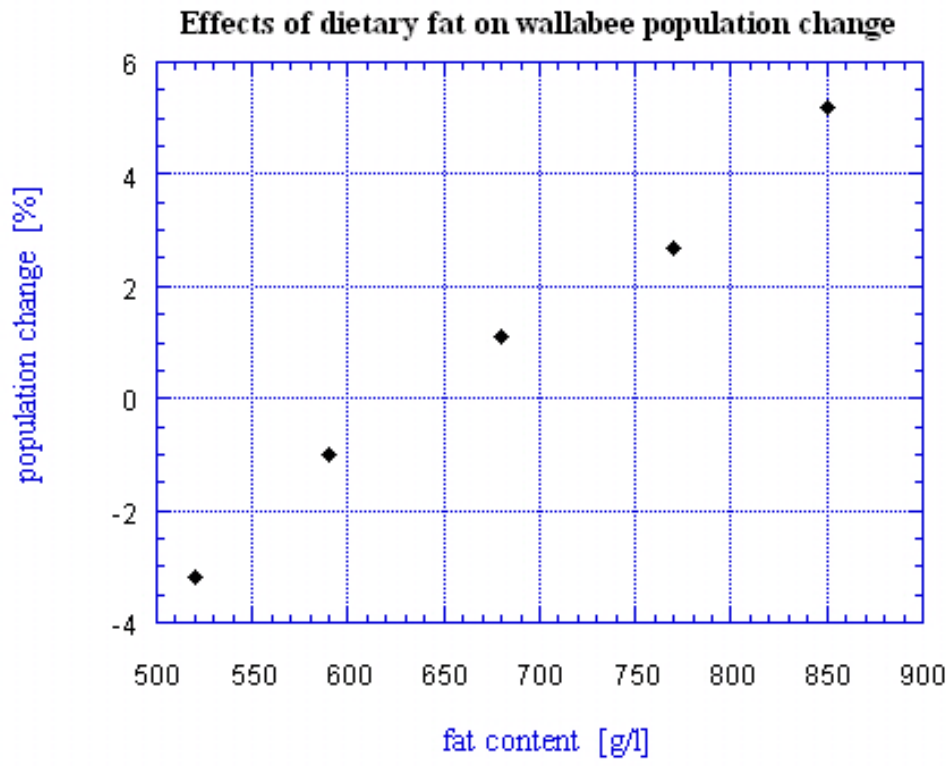

FIG. 2: How to plot data

The parameter a is called the *slope* of the line and the parameter b is called the *intercept*. Oftentimes the physics information in an experiment can be extracted by determining these two parameters. Doing so requires two steps:

1. *Draw the "best" straight line through the data.* The best straight line through your data is *not* a line through the endpoints. In fact, the best straight line may not even go through *any* of your data points. The best straight line is the line that, on average, is closest to all of the points. There are many ways to interpret this last statement. Most computer programs interpret it by performing a least squares regression on the data; the least squares regression defines "closest on average" as the line that minimizes the sum of the squared distance between the data and the line. For this lab you will be making all your graphs *by hand*, so you will draw the best straight line *by eye*.

2. *Determine the slope.* The slope of a straight line is defined as the ratio of the rise —or change along the ordinate axis— to the run —change along the abscissa— for any segment of the line. The larger a line segment you use, the more accurate will be your determination of the slope. Remember that the units of the rise and run are the same as the units used on the ordinate and abscissa axes, respectively.

Figure 3 shows a best straight line through the wallabee population data, along with a calculation of the slope.

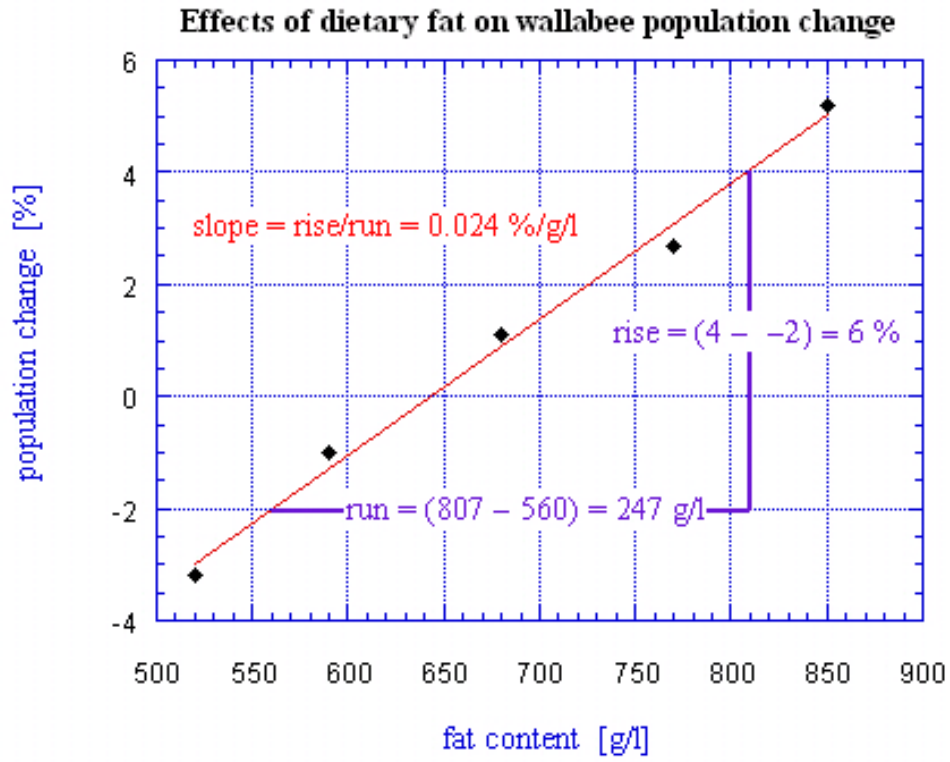

FIG. 3: How to find the slope of a straight line

Experimental Procedure

1. With a stopwatch, time the fall of a crumpled paper wad from a fixed height. Repeat the measurement five times from the same height. Tabulate your data.

2. Measure the length of five pieces of plastic pipe. Choose pieces whose lengths are representative of the entire collection. Use the digital calipers.

3. Find the mass of each pipe whose length you measured in Step 2, using the mass balance. Tabulate all mass and length measurements.

4. Calculate and report the average time of fall for your measurements in Step 1. Also find the rms deviation for your measurements, and the standard deviation of the mean for the time of fall. Finally, report the 95% confidence interval for this time as determined by your experiment.

5. Plot the mass vs. length for the plastic pipe. Find the slope of the best straight line through your data. The ordinate intercept of this line should be zero (why?), so be sure your 'best fit' line passes through the origin. Briefly discuss the physical significance of the slope you have found. Could the 'fit' be improved if the ordinate intercept were not forced to zero? What is the problem with doing this?