Physics 201 Lab: Measuring the Acceleration due to Gravity

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Free Fall

Free fall refers to a special case of straight-line motion where the acceleration is directed vertically downward with a constant magnitude, usually denoted by g. This acceleration is imparted to all objects near the surface of the Earth, and is caused by the 'tug' of the Earth's gravity; accordingly, g is referred to as the acceleration due to gravity.

But g is not really constant. For one thing, the density of the Earth is not uniform, creating small local variations in the Earth's gravity pull. In addition, g depends on the elevation of the object above the Earth's surface. g also varies with latitude, an effect that can be traced to the Earth's rotation on its axis. All these effects are small and wholly negligible in many applications. The accepted value for g in our locale is (to 4-digit accuracy) 9.797 m/s^2.

In this experiment we are going to measure the local value of g by dropping a slotted plate through a photogate timer. The timer will give us a series of measurements for the velocity v of the plate as a function of the vertical position y. Recall that we have two kinematic equations to describe the motion of an object with constant acceleration. For acceleration equal to g, they are:

$$y = y_0 + v_0 t + \frac{1}{2} g t^2 \tag{1}$$

and

$$v = v_0 + gt \tag{2}$$

The equation we actually require derives from these two by eliminating the time t between them. Solving Equation 2 for t and inserting the result into Equation 1 gives (after some manipulation),

$$v^2 = v_0^2 + 2g(y - y_0) \tag{3}$$

Inspection of Equation 3 reveals two facts pertinent to our analysis. The first is that only the vertical displacement $\Delta y = y - y_0$ matters, so that we can place the origin of the y-axis anywhere that is convenient. The second is that if we were to plot v^2 vs Δy , the graph should be a straight line with slope equal to $2g$ and [ordinate] intercept equal to v_0^2.

Procedure

A schematic diagram of the experimental setup is shown in Figure 1A. The slotted plate is dropped vertically through the photogate timing device. The timing module will be set to the *s2* functional mode. In this mode of operation, a timing cycle begins when the photogate beam is interrupted and ends upon being interrupted again. The next cycle begins when the beam is once again interrupted, and so on. The plate, depicted in Figure 1B, consists of a series of 1 cm wide slots, each 1 cm apart. The distance over which a timing cycle occurs is therefore 2 cm. Because a new cycle doesn't begin until the beam is again interrupted, the timing cyles themselves correspond to plate regions separated by 4 cm, as shown in the figure.

Prior to conducting the experiment, you should verify that the timer function mode is set to $s2$. The function mode can be altered by repeatedly pushing the function button. The device cycles through each of the functions; an LED indicates which function is activated. There is also an LED indicator that identifies the units in which the result will be output on the display panel. You should press the *clear* button, to clear out any previous results. After dropping the plate through the timing gate, press the *stop* button.

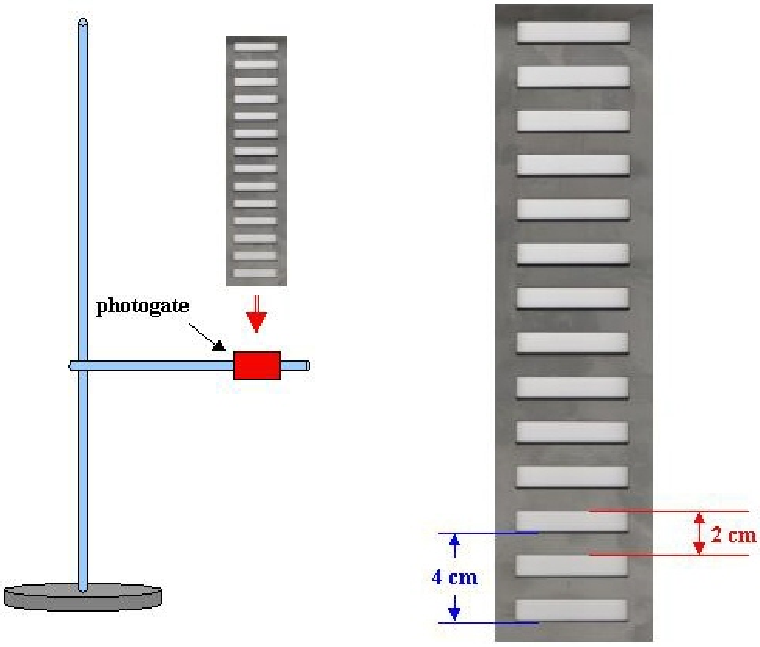

Figure 1A Figure 1B

FIG. 1: Physical setup and dimensions for the measurement of g.

The display unit will then cyclically read out the time intervals. Preceding each time interval readout, the display will indicate which time interval is about to be shown; i.e., $1^{\text{st}}, 2^{\text{nd}}, 3^{\text{rd}}, \ldots$. You should record these times, starting from #1 through #7. This is your raw data.

The average velocity in the j^{th} time interval Δt_j is

$$v_j = \frac{2w}{\Delta t_j} \tag{4}$$

where $2w = 2$ cm is the displacement of the plate during this interval ($w = 1$ cm is the width of both the slot and the metal strip separating adjacent slots, cf. Figure 1B). Since the acceleration is constant, the average velocity of Equation 4 is numerically equal to the *instantaneous* velocity of the plate at the *mid-time* of the timing cycle. But what is the displacement of the plate at this instant? For the very first timing cycle, this displacement is marked Δy_1 in Figure 2. If the plate falls uniformly, it would fall half the distance in half the time, so Δy_1 would be just $w = 1$ cm. But for accelerated motion it must be less than that. The actual displacement of the plate during the first half of the first timing cycle can be expressed as (cf. Equations 1, 2, and 4)

$$\begin{aligned}
\Delta y_1 &= v_0 \frac{\Delta t_1}{2} + \frac{1}{2} g (\frac{\Delta t_1}{2})^2 \\
&= v \frac{\Delta t_1}{2} - \frac{1}{2} g (\frac{\Delta t_1}{2})^2 \\
&= w - g \frac{(\Delta t_1)^2}{8}
\end{aligned}$$

The second term on the right in the last line is the correction for accelerated motion. For the j^{th} timing cycle, the same correction applies, but with Δt_1 replaced by Δt_j. These corrections are small for short time intervals (the largest of which is also the first). The fractional discrepancy introduced by ignoring this correction is

$$\frac{g}{w}\frac{(\Delta t_1)^2}{8} = \frac{gw}{2(v_1)^2} \tag{5}$$

Evidently $\sqrt{gw}$ is a velocity, the value of which for our setup is about 33 cm/s. If v_1 exceeds this by a factor of 10, the correction is at the 0.5% level. With this in mind, we will ignore the acceleration correction for *all* timing cycles. The result is a simple rule for finding Δy_j: starting with $\Delta y_1 = w$, we simply add $4w$ ($= 4$ cm) to obtain Δy_2, then add another $4w$ to obtain Δy_3, etc. (cf. Figure 2).

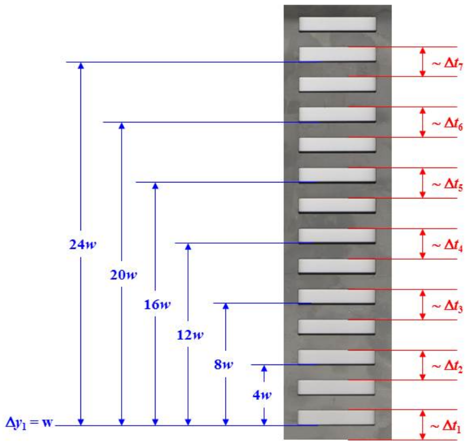

Figure 2

FIG. 2: Coordinate system for analyzing data. Red lines on the right delineate sections of the plate that are 'active' during the indicated timing interval. Blue lines along the left edge mark the locations associated with the instantaneous velocities v_j of Equation 4, assuming negligible acceleration during each timing cycle.

The complete procedure is as follows:

1. Set up the timing device so that the functional mode is $s2$. Clear any previous data.

2. *Carefully* (a steady hand is required!) drop the slotted plate vertically through the photogate, then press the *stop* button on the timing module. Record the time intervals Δt_j. Pay attention to units.

3. Calculate the velocities v_j using Equation 4.

4. Tabulate the corresponding displacements Δy_j (from Figure 2).

5. In Excel, make a scatter plot of v_j^2 vs Δy_j and perform a least-squares [linear] fit to the plotted points. Include on your graph the equation of the trendline and the correlation coefficient R^2 for the fit. [Values for R^2 close to unity signal a good fit to the data.]

6. Use Excel's LINEST function[1] to find the slope of the 'best-fit' line, and the standard error in the slope. The slope information populates the first column of the LINEST output block (slope in the first row; standard error in the second row). Deduce from this an experimental value for g along with the standard error in this value, σ. Report your result in the form $g \pm \sigma$. Does the accepted value of g for this locale fall within your reported range? If not, offer suggestions as to why not.

7. LINEST reports the ordinate intercept for the 'best-fit' line (and the standard error of the intercept) in the second output column. From the intercept, extract a characteristic velocity for your experiment. What is the physical significance of this velocity?

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[1] Since LINEST is an *array function* (a single function call generates multiple outputs), the function call must be made in a special way: Select the cell containing the LINEST entry. Starting with the formula cell, highlight a block of cells (2 columns by 3 rows will suffice) to receive the output values, then press F2 (fn+F2 on Mac). Finally, press Ctrl+Shift+Enter. More details on using LINEST are available in Excel Help.