Physics 201 Lab: Measuring the Coefficient of Kinetic Friction

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Theoretical Discussion

When two surfaces in contact move relative to one another in a direction tangent to the surfaces, each surface exerts a shearing force on the other known as kinetic friction. Although complicated in origin, this force satisfies a simple mathematical relationship,

$$f_k = \mu_k N$$

where f_k, the kinetic friction force, is exerted tangent to the surface and opposite to its direction of motion. Here N is the force that one surface exerts normal to the other surface, and μ_k, the coefficient of kinetic friction, depends only on the intrinsic properties of the two surfaces in contact.[1]

The purpose of this experiment is to determine the coefficient of kinetic friction between the bottom surface of a wooden box, lined with felt, and a glass plate positioned on an inclined plane; the experimental setup is shown schematically in Figure 1.

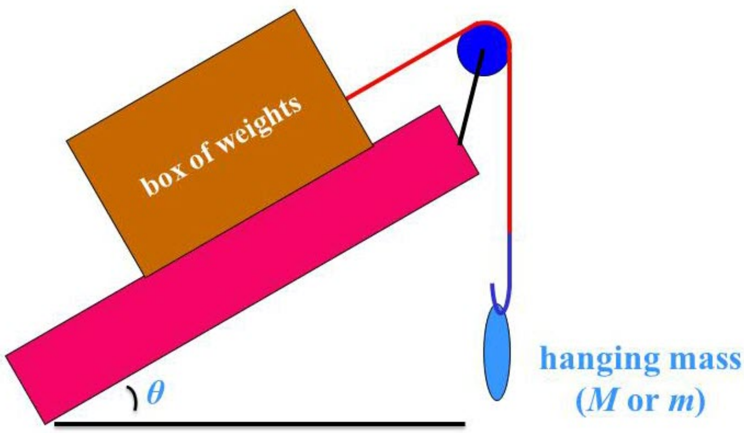

FIG. 1: Experimental setup for determining the coefficient of kinetic friction.

Given the block positioned on the plane inclined at angle θ, we denote by M the amount of mass suspended from the string, such that when the box – initially at rest – is nudged *up* the incline, it moves with constant velocity. Correspondingly, let m be the amount of mass attached to the string, such that when the box is nudged *down* the incline, it moves with constant velocity down the incline. This very situation was the subject of a lecture Example; with some obvious changes in notation, the results are

$$\begin{aligned} Mg &= m_b g(\sin\theta + \mu_k \cos\theta) \\ mg &= m_b g(\sin\theta - \mu_k \cos\theta) \end{aligned}$$

where m_b denotes the mass of the sliding box (including its contents). Adding and subtracting these equations gives the new pair

$$
\begin{aligned}
M + m &= 2m_b \sin\theta \\
M - m &= 2m_b \mu_k \cos\theta
\end{aligned}
$$

Finally, dividing the second of these equations by the first eliminates m_b, leaving

$$
\frac{M - m}{M + m} = \mu_k \cot\theta \tag{1}
$$

Equation 1 implies that the graph of $\frac{M-m}{M+m}$ (ordinate) vs. $\cot\theta$ (abscissa) is a straight line with slope μ_k. Thus, by finding the minimum and maximum values for the hanging mass that lead to equilibrium (constant velocity) for a number of different elevation angles θ, we have the means to experimentally determine the kinetic friction coefficient between the box and the incline.

Experimental Procedure

1. Place a 500 gram mass and two 100 gram masses in the box, as shown in Figure 2.
2. Without the mass holder attached, determine the angle of elevation of the plane such that when the box is nudged down the incline, it moves with constant velocity (see Tips & Tricks below). Since there is no mass attached to the string, this corresponds to $m = 0$. Next, attach the mass holder to the string and add a sufficient amount of mass so that when the box is nudged up the incline, it moves with constant velocity; this is the mass M for this case. Use the digital level to determine a precise value for the elevation angle θ, and record that value, along with the value you found for M (remember to include the mass of the hanger).
3. Raise the plane approximately $5°$ above the angle obtained in the previous step. Determine M and m to achieve uniform motion up and down the plane, respectively (cf. Tips & Tricks). Record your measured values for M, m, and θ.
4. Repeat the previous step for three additional $5°$ increments.
5. Using Excel, plot a graph of $\frac{M-m}{M+m}$ (ordinate) vs. $\cot\theta$ (abscissa). You should have 5 data pairs on your graph. (Note: since the first measurement corresponds to $m = 0$, the mass ratio for this case evaluates to one, independent of the value of M.)
6. Perform a linear regression of the data, forcing the intercept to be zero. Report the values of the estimated regression parameters and the value of R^2.
7. Use Excel's LINEST function[2] to find the slope of your graph (μ_k) and the standard error in the slope (σ). [Note: the first logical argument in the LINEST function call must be FALSE here, to force an intercept of zero.] Report your results in the form $\mu_k \pm \sigma$, corresponding to a 68% confidence interval.

Experimental Tips & Tricks

In performing the experiment you are instructed to nudge the box so that it moves with constant velocity. To obtain acceptable experimental results, this procedure should be performed in a consistent way. Good practice dictates that proper attention be paid to the following:

1. Locate the box at a suitable position on the plane. This position should be used throughout the experiment, since the intrinsic properties of the plane on which the coefficient of kinetic friction depends are, in general, not homogeneous along the surface of the plane.
2. Press the box gently against the plane to insure that it is making good contact with the plane.

FIG. 2: Box and Weights. The 500-gram and two 100-gram masses are placed in the box as shown.

3. Give the box an abrupt – but not excessively hard – push in the direction in which it is desired that the box should move (the box should not acquire a large initial speed). If, after the initial push, the speed of the box slows quickly to the point where the box creeps along for an additional distance of approximately 6 cm or more, then the condition of constant velocity can be assumed satisfied.

4. Lastly, when adding mass to the mass holder, do so in increments of 20 g or larger.

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[1] Friction, although it has been observed and described mathematically at an elementary level for several hundred years, is nonetheless very complex and is not, even today, completely understood at the most fundamental, microscopic level.

[2] Since LINEST is an *array function* (a single function call generates multiple outputs), the function call must be made in a special way: Select the cell containing the LINEST entry. Starting with the formula cell, highlight a block of cells (2 columns by 3 rows will suffice) to receive the output values, then press F2 (fn+F2 on Mac). Finally, press Ctrl+Shift+Enter. More details on using LINEST are available in Excel Help.