MATH 261 **Sample** Final Exam

Simplify answers. No work, no credit | Name:

Score			
1	11		
2	12		
3	13		
4	14		
5	15		
6	16		
7	17		
8	18		
9	19		
10	20		
Tot			

1. Given a force $\mathbf{F} = 5\mathbf{i} - 3\mathbf{j}$ and a position vector $\mathbf{r} = 2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, compute:

a) The torque $\mathbf{r} \times \mathbf{F}$.

Ans:_____.

b) $|\mathbf{r} \times \mathbf{F}|$.

Ans:_____.

2. Given the point $P(1, -1, 5)$ and the line $L: \ \mathbf{r}(t) = \langle 3 - t, 3 + 2t, 2 + 5t \rangle$, find:

a) The direction vector of the line.

Ans:_____.

b) The line through P and $\|$ to L.

Ans:_____.

3. Write parametric equations for:

a) The hyperbola $x^2 - y^2 = 9$.

Ans:_____.

b) The cone $x^2 + y^2 - z^2 = 0$.

Ans:_____.

4. Given the surface $\mathcal{S}: \ z = xy - 12$ and the point $P(2, 2, -8)$

a) A normal to $\mathcal{S}$ at P.

Ans:_____.

b) The equation of the tangent plane at P.

Ans:_____.

5. A particle moves along the path $\mathbf{r}(t) = \sqrt{2}t\ \mathbf{i} + e^t\ \mathbf{j} + e^{-t}\ \mathbf{k}$. Find the:

a) Speed at (0,1,1).

Ans:_____.

b) Acceleration.

Ans:_____.

6. c) Tangential acceleration at (0,1,1).

Ans:_____.

d) Centripetal acceleration at (0,1,1).

Ans:_____.

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7. Suppose $f(x,y) = \ln(x^2+y^2)$.

a) Compute ∇f.

Ans:_____.

b) Compute df.

Ans:_____.

8. Let $f(x,y) = x^4y^3 - 3x^2 + y$ and $\mathbf{u}$ point in the direction given by the angle $\theta = \pi/6$. Find

a) The unit direction vector $\mathbf{u}$.

Ans:_____.

b) $D_{\mathbf{u}}f(1,1)$.

Ans:_____.

9. Let $z = f(x,y) = xy - x^2 - y^2 - 2x - 2y + 7$.

a) Find the critical points.

Ans:_____.

b) Classify the critical points.

Ans:_____.

10. Let $xe^z + y^2z^3 = 6$. Use implicit differentiation to find:

a) z_x.

Ans:_____.

b) z_y.

Ans:_____.

11. A plate of density $\rho = 5$ is bounded by $y = \sqrt{a^2 - x^2}$ and $y = 0$. Set up the integrals for

a) $y_{c.m}$

b) I_z

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12. Setup and compute $\int\int dS$, where S is the part of the surface $z = xy + 4$, bounded by $x^2 + y^2 = 4$.

Ans:_____.

13. Let $I = \int_0^3 \int_{y^2}^9 2ye^{x^2}\, dx\, dy$.

a) Reverse the order of integration

Ans:_____.

b) Evaluate the integral

Ans:_____.

14. Find the volume bounded by $z = 49 - x^2 - y^2,\ z \geq 0$, and $x^2 + y^2 = 9$.

Ans:_____.

15. Find the volume of spherical cap of the sphere $x^2 + y^2 + z^2 = 2$ above the plane z=1.
Hint: the volume of a cone is $V_c = \frac{1}{3}\pi r^2 h$

Ans:_____.

16. True or False?

___. a) If $\nabla \cdot \mathbf{F} = 0$, then $\mathbf{F}$ is conservative.
___. b) If $\mathbf{F} = \nabla \times \mathbf{G}$, then $\mathbf{F}$ is incompressible.
___. c) If $\int_C \mathbf{F} \cdot d\mathbf{r}$ is path independent, then $\nabla \times \mathbf{F} = 0$.
___. d) If $\nabla \times \mathbf{F} = 0$, then $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$.
___. e) If $\mathrm{bd}(S_1) = \mathrm{bd}(S_2)$ then $\int\int_{S_1} \mathbf{F} \cdot d\mathbf{S} = \int\int_{S_2} \mathbf{F} \cdot d\mathbf{S}$

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17. Let $\mathbf{F} = (e^x \cos y + yz)\,\mathbf{i} + (xz - e^x \sin y)\,\mathbf{j} + (xy + z)\,\mathbf{k}$ and C is the line segment joining (0,0,0) to $(-1, \pi/2, -2)$

a) Find f such that $\mathbf{F} = \nabla f$.

Ans:_____.

b) Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$

Ans:_____.

18. Compute: $\oint_C -y^3\,dx + x^3\,dy$, where C is the circle $x^2 + y^2 = 36$

Ans:_____.

19. Find $\int\int (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$, where $\mathbf{F} = \langle 6y + z^2, 2x - y^3, 5y^2 z \rangle$ and $S: \quad z = 36 - 4x^2 - 4y^2$ with $z \geq 0$.

Ans:_____.

20. Find: $\int\int_\Sigma \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = 3z^2 x\mathbf{i} + y^3\mathbf{j} + 3x^2 z\mathbf{k}$, and Σ is bounded by $x = \sqrt{16 - z^2 - y^2}$ and $x = 0$.

Ans:_____.

Extra Space

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