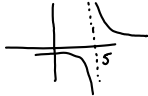


MATH 161 EXAM 1, Fall 2003

Show all work!		Name:	Score																
1.	<p>The displacement of a particle is given by $s = 4t^2$. Find the average velocity in the interval:</p> <p>a) [1, 3]</p> <table border="1" style="display: inline-table; vertical-align: middle;"> <tr><td>t</td><td>s</td></tr> <tr><td>1</td><td>4</td></tr> <tr><td>3</td><td>36</td></tr> </table> $v_{Ave} = \frac{36-4}{3-1} = 16$	t	s	1	4	3	36	<p>b) [1, 1.2]</p> <table border="1" style="display: inline-table; vertical-align: middle;"> <tr><td>t</td><td>s</td></tr> <tr><td>1</td><td>4</td></tr> <tr><td>1.2</td><td>5.76</td></tr> </table> $v_{Ave} = \frac{5.76-4}{1.2-1} = 8.8$	t	s	1	4	1.2	5.76	1				
t	s																		
1	4																		
3	36																		
t	s																		
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			6																
			7																
			8																
2.	<p>Guess the limit. Write one sentence explaining how you obtained your answer.</p> <p>a) $\lim_{x \rightarrow 0} (\sin x)/x$</p> <table border="1" style="display: inline-table; vertical-align: middle;"> <tr><td>x</td><td>(sin x)/x</td></tr> <tr><td>± 0.1</td><td>0.99833</td></tr> <tr><td>± 0.01</td><td>0.99998</td></tr> <tr><td>± 0.001</td><td>1</td></tr> </table> <p>• Used the TABLE feature of TI 83 • Graphed</p> <p>Ans: <u>1</u></p>	x	(sin x)/x	± 0.1	0.99833	± 0.01	0.99998	± 0.001	1	<p>b) $\lim_{x \rightarrow 0} (x \ln x)$</p> <table border="1" style="display: inline-table; vertical-align: middle;"> <tr><td>x</td><td>x ln x</td></tr> <tr><td>0.1</td><td>-0.2303</td></tr> <tr><td>0.01</td><td>-0.0461</td></tr> <tr><td>0.001</td><td>-0.0069</td></tr> </table> <p>Graphed and evaluated near $x=0$</p> <p>Ans: <u>0</u></p>	x	x ln x	0.1	-0.2303	0.01	-0.0461	0.001	-0.0069	9
x	(sin x)/x																		
± 0.1	0.99833																		
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0.1	-0.2303																		
0.01	-0.0461																		
0.001	-0.0069																		
			10																
			Tot																
3.	<p>Determine the infinite limit:</p> <p>a) $\lim_{x \rightarrow 5^+} \frac{2}{x-5}$</p> <p>As $x \rightarrow 5^+$ (x-5) becomes small so $\frac{2}{(x-5)}$ becomes large</p>  <p>Ans: <u>∞</u></p>	<p>b) $\lim_{x \rightarrow \infty} \frac{3x^3 - 8}{2 + x - 5x^3} = \lim_{x \rightarrow \infty} \frac{3 - (8/x^3)}{(2/x^3) + (1/x^2) - 5}$</p> $= \frac{3-0}{0+0-5} = -\frac{3}{5}$ <p>Ans: <u>$-\frac{3}{5}$</u></p>																	
4.	<p>Evaluate the limit and justify every step using the Limit Law(s).</p> <p>a) $\lim_{x \rightarrow 3} (2x^2 - 5) = \lim_{x \rightarrow 3} 2x^2 - \lim_{x \rightarrow 3} 5$</p> $= 2 \lim_{x \rightarrow 3} x^2 - 5$ $= 2 [\lim_{x \rightarrow 3} x]^2 - 5$ $= 2(3)^2 - 5 = 13$ <p>Ans: _____</p>	<p>b) $\lim_{x \rightarrow 5^-} \sqrt{25 - x^2} = \sqrt{\lim_{x \rightarrow 5^-} (25 - x^2)}$</p> $= \sqrt{\lim_{x \rightarrow 5^-} 25 - \lim_{x \rightarrow 5^-} x^2}$ $= \sqrt{25 - (\lim_{x \rightarrow 5^-} x)^2}$ $= \sqrt{25 - 25} = 0$ <p>Ans: _____</p>																	
5.	<p>Evaluate the limit, if it exists:</p> <p>a) $\lim_{x \rightarrow 3} \frac{x^2 - 3x}{x^2 + 2x - 15} = \lim_{x \rightarrow 3} \frac{x(x-3)}{(x-3)(x+5)}$</p> $= \lim_{x \rightarrow 3} \frac{x}{x+5} = \frac{3}{8}$ <p>Ans: _____</p>	<p>b) $\lim_{h \rightarrow 0} \frac{(1+h)^2 - 1}{h} = \lim_{h \rightarrow 0} \frac{1}{h} [1 + 2h + h^2 - 1]$</p> $= \lim_{h \rightarrow 0} \frac{1}{h} (2h + h^2)$ $= \lim_{h \rightarrow 0} (2 + h) = 2$ <p>Ans: _____</p>																	
Extra Space																			

Part II.	Name:																				
<p>6. Using the ϵ, δ definition, prove that: $\lim_{x \rightarrow 10} (4 - x/5) = 2$</p> <p><i>Given $\epsilon > 0$, choose $\delta = 5\epsilon$. Assume that $x - 10 < \delta$, that is $x - 10 < 5\epsilon$</i></p> <p><i>Then, $f(x) - L = 4 - \frac{x}{5} - 2$</i></p> $= 2 - \frac{x}{5} = \frac{1}{5} 10 - x $ $= \frac{1}{5} x - 10 $ $< \frac{1}{5} \cdot 5\epsilon$ $< \epsilon$ <p style="text-align: right;"><i>Q.E.D.</i></p>																					
<p>7. a) Define precisely what is meant by a continuous function.</p> <p><i>$f(x)$ is continuous at $x = a$ if</i></p> <p>a) <i>$f(a)$ is defined</i></p> <p>b) <i>$\lim_{x \rightarrow a} f(x)$ exists</i></p> <p>c) <i>$\lim_{x \rightarrow a} f(x) = f(a)$</i></p>	<p>b) Is $f(x)$ continuous at $x = 2$?, Explain!</p> $f(x) = \begin{cases} x+2 & x \leq 2 \\ x^2 & x > 2 \end{cases}$ <p>i) $f(2) = 2+2 = 4$ <i>defined</i></p> <p>ii) $\lim_{x \rightarrow 2^+} f(x) = 2^2 = 4$</p> <p>$\lim_{x \rightarrow 2^-} f(x) = 2+2 = 4$</p> <p>$\Rightarrow \lim_{x \rightarrow 2} f(x) = 4$ <i>exists</i></p> <p>iii) $\lim_{x \rightarrow 2} f(x) = f(2) = 4$ ✓</p> <p style="text-align: right;"><i>Yes</i></p>																				
<p>8. Use a limit method to find the slope of the tangent line to $f(x)$ at the point $(4, 1)$.</p> $f(x) = \frac{6}{x+2}$ $f'(x) = \lim_{x \rightarrow 4} \frac{f(x) - f(4)}{x - 4}$ $= \lim_{x \rightarrow 4} \frac{1}{x-4} \left[\frac{6}{x+2} - 1 \right]$ $= \lim_{x \rightarrow 4} \frac{1}{x-4} \left[\frac{6 - (x+2)}{x+2} \right]$ $= \lim_{x \rightarrow 4} \frac{1}{x-4} \left[\frac{4-x}{x+2} \right] = \lim_{x \rightarrow 4} \frac{1}{x+2} = -\frac{1}{6}$ <p style="text-align: center;">Ans: _____</p>	<p style="text-align: center;">Ans: _____</p>																				
<p>9. The price $P(t)$ per share of a stock for 9 consecutive days is given by:</p> <table border="1" style="width: 100%; text-align: center;"> <tr> <td>t</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> <td>8</td> <td>9</td> </tr> <tr> <td>P</td> <td>2.28</td> <td>4.31</td> <td>5.80</td> <td>8.12</td> <td>10.44</td> <td>11.31</td> <td>13.97</td> <td>15.01</td> <td>17.64</td> </tr> </table> <p>a) Estimate the value of $P'(5)$.</p> $P'(5) \approx \frac{11.31 - 8.12}{6 - 4} = 1.60$ <p style="text-align: center;">Ans: _____</p>	t	1	2	3	4	5	6	7	8	9	P	2.28	4.31	5.80	8.12	10.44	11.31	13.97	15.01	17.64	<p>b) What is the meaning of this value.</p> <p><i>At $t = 5$ the price is rising at the rate of \$1.60/day</i></p> <p style="text-align: center;">Ans: _____</p>
t	1	2	3	4	5	6	7	8	9												
P	2.28	4.31	5.80	8.12	10.44	11.31	13.97	15.01	17.64												
<p>10. Use the limit definition to find the derivative of $f(x) = \sqrt{x}$</p> $f'(x) = \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$ $= \lim_{h \rightarrow 0} \frac{1}{h} (\sqrt{x+h} - \sqrt{x}) \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$ $= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{x+h-x}{\sqrt{x+h} + \sqrt{x}} \right]$ $= \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$ <p style="text-align: center;">Ans: _____</p>	<p style="text-align: center;">Ans: _____</p>																				
Extra Space																					