MATH 261 Final Exam, Spring 2005

Simplify answers. No work, no credit | Name:

Score			
1	11		
2	12		
3	13		
4	14		
5	15		
6	16		
7	17		
8	18		
9	19		
10	20		
Tot			

1. Given a triangle with vertices at $P(2,1,1)$, $Q(-1,1,5)$ and $R(3,3,2)$, find:
   a) The length of the base $\overline{PQ}$.
   Ans:_____.
   b) The angle $\angle RPQ$.
   Ans:_____.

2. Given the plane $\mathcal{P} : 3x - 2y + z = 4$ and the point $Q(3,-2,8)$, find the equation of :
   a) The line normal to $\mathcal{P}$.
   Ans:_____.
   b) The plane $\|$ to $\mathcal{P}$ which contains Q.
   Ans:_____.

3. A plane $\mathcal{P}$ contains the line $\mathbf{r}(t) = \langle 2+t, t, 3-4t\rangle$ and the point $P(3,3,-1)$. Find:
   a) The normal to $\mathcal{P}$.
   Ans:_____.
   b) The equation of $\mathcal{P}$.
   Ans:_____.

4. Change from cylindrical to Cartesian coordinates and identify the surface:
   a) $z = 4 - r^2$
   Ans:_____.
   b) $r = 5\cos\theta$.
   Ans:_____.

5. A particle moves along the path $\mathbf{r}(t) = t^3\,\mathbf{i} + t\,\mathbf{j} + \frac{1}{2}\sqrt{6}t^2\,\mathbf{k}$, Find:
   a) The speed and acceleration at $t = 1$.
   Ans:_____.
   b) The arc length for $1 \le t \le 3$.
   Ans:_____.

6. Describe the level surfaces of $f(x,y,z) = 4x^2 - 9y^2 + z^2$ for:
   a) $k = 0$.
   Ans:_____.
   b) $k = 36$.
   Ans:_____.

Extra Space

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7. Suppose $\mathbf{F} = e^{xy}\mathbf{i} - \cos y\mathbf{j} + \sin^2 z\mathbf{k}$.

a) Compute $\nabla \times \mathbf{F}$.

Ans:_____.

b) Compute $\nabla \cdot \mathbf{F}$.

Ans:_____.

8. Let $z = f(x^2 + y^2)$. Use the chain rule to show that: $y\frac{\partial z}{\partial x} - x\frac{\partial z}{\partial y} = 0$.

9. Let $f(x, y) = 3x^2 - 2xy + y^2 - 8y$.

a) Find the critical points.

Ans:_____.

b) Find the maxima, minima and saddle points.

Ans:_____.

10. Let $e^{xz} + y^2 + 5z = 1$. Find:

a) dz.

Ans:_____.

b) z_x.

Ans:_____.

11. Consider the sphere $\mathcal{S}: \ x^2 + y^2 + z^2 = 25$ and the point $P(-3, 0, 4)$. Find:

a) The unit normal to $\mathcal{S}$ at P.

Ans:_____.

b) the equation of the tangent plane to $\mathcal{S}$ at P.

Ans:_____.

Extra space

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12. A plate of density $\rho = y^2$ is bounded by $y = 4 - x^2$ and $y = 0$. Set up and evaluate (using a CAS) the integrals for:

a) $y_{c.m}$

Ans:_____.

b) I_x

Ans:_____.

13. Let $I = \int_0^3 \int_{y/3}^1 e^{-x^2}\, dx\, dy$.

a) Reverse the order of integration

Ans:_____.

b) Evaluate the integral in part (a).

Ans:_____.

14. A tetrahedron is bounded by the coordinate planes and by the plane $4x + 2y + z = 4$. Set the volume integral as a:

a) Double integral.

Ans:_____.

b) Triple integral.

Ans:_____.

15. The transformation $u = ax,\ v = by$ maps the ellipse $\frac{u^2}{a^2} + \frac{v^2}{b^2} = 1$ into the circle $x^2 + y^2 = 1$. Use the change of variables to compute the area of the ellipse.

a) Find the Jacobian and $dA = |\frac{\partial(u,v)}{\partial(x,y)}| dxdy$.

Ans:_____.

b) Compute $\int\int\ dA$ using polar coordinates.

Ans:_____.

16. Find the surface area of the part of the paraboloid $x = 25 - y^2 - z^2$ bounded by the cylinder $y^2 + z^2 = 9$.

a) Setup the integral.

Ans:_____.

b) Evaluate the integral.

Ans:_____.

17. Evaluate $\int_c \nabla f \cdot d\mathbf{r}$, where $f(x, y, z) = xy^2z^3$, and $\mathbf{r}(t) = t\,\mathbf{i} + (t^2 + t)\,\mathbf{j} + \sin(3\pi t/2)\,\mathbf{k}$. ($0 \le t \le 1$.)

Ans:_____.

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18. Use Stoke's theorem to compute $\oint_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = z^2\,\mathbf{i} + 2x\,\mathbf{j} - y^3\,\mathbf{k}$ and C is the boundary of the paraboloid $z = 4 - x^2 - y^2$, $z \geq 3$, with upward orientation.

Ans:_____.

19 Find $\int\int_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = (x^3 + 5z)\,\mathbf{i} + y^3\,\mathbf{j} + 3x\,\mathbf{k}$ and S is the surface of the solid bounded by the cylinder $x^2 + y^2 = 9$, and the planes $z = 0$ and $z = 2$.

Ans:_____.

20. Find $\int\int_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$, where $\mathbf{F} = (-5z + x^5)\,\mathbf{i} + \ln y\,\mathbf{j} + (4x + e^z)\,\mathbf{k}$, and S is the surface $9x^2 + y^2 + 9z^2 = 25$ with $y \geq 4$.

Ans:_____.

Extra Space