

4.10 Antiderivatives

Definition: $F(x)$ is an antiderivative of $f(x)$ if $F'(x) = f(x)$.

Any two antiderivatives of $f(x)$ differ at most by a constant. So, if $F(x)$ is an antiderivative of $f(x)$, the most general antiderivative is $F(x) + C$. C - arbitrary const.

Examples: Find the most general antiderivative of the given functions

1. $f(x) = 3x^2 + 5x - 2$ $F(x) = x^3 + \frac{5}{2}x^2 - 2x + C$

2. $f(x) = 6\sqrt{x} - \sqrt[6]{x}$ $F(x) = 6 \cdot \frac{2}{3} x^{3/2} - \frac{6}{7} x^{7/6} + C$
 $= 4x\sqrt{x} - \frac{6}{7} x \sqrt[6]{x} + C$

3. $f(x) = 4e^x - \sec^2 x$ $F(x) = 4e^x - \tan x + C$

4. $f(\theta) = \frac{\sin \theta}{\cos^2 \theta}$ $f(x) = \frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\cos \theta} = \tan \theta \sec \theta$
 $F(x) = \sec \theta + C$

To evaluate the constant C one must be given conditions.

Ex Find $f(x)$

5. $f'(x) = 1 - 6x$
 $f(0) = 8$

$f(x) = x - 3x^2 + C$
 $f(0) = C = 8 \Rightarrow f(x) = x - 3x^2 + 8$

6. $f''(x) = 2 - 12x$
 $f(0) = 3$
 $f'(0) = 2$

$f'(x) = 2x - 6x^2 + C = 2x - 6x^2 + 2$
 $f(x) = x^2 - 2x^3 + 2x + D$
 $= x^2 - 2x^3 + 2x + 3$

FREE FALL

An object is thrown downwards from a height s_0 with initial velocity v_0 . What is the height as a function of time?

Solution

Newton's Second Law

$F = ma$

$s(0) = s_0$

$v(0) = v_0$

$m \frac{dv}{dt} = mg$

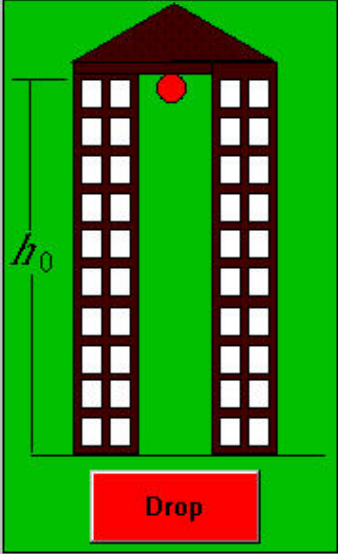
$\frac{dv}{dt} = g$

$v = gt + v_0$

$\frac{ds}{dt} = gt + v_0$

$s = \frac{1}{2}gt^2 + v_0t + s_0$

Graph



If one follows the convention that g is positive ($g = 9.8 \text{ m/s}^2$) but gravity points down, then the free fall equations become

$$v = -gt + v_0$$

$$s = -\frac{1}{2}gt^2 + v_0t + s_0$$

*In USA units
 $g = 32 \text{ ft/s}^2$*

4.10b Antiderivatives. (Cont)

Example: A stone was dropped off a cliff and hits the ground with a speed of 120 ft/s. What was the height of the cliff?

$$v_0 = 0, \quad v = -gt + v_0 \quad -120 = -16t$$

$$\quad \quad \quad = -gt \quad \quad \quad t = \frac{120}{16} = \frac{15}{2}$$

$$s = -\frac{1}{2}gt^2 + s_0$$

$$0 = -\frac{1}{2}16\left(\frac{15}{2}\right)^2 + s_0$$

$$s_0 = \frac{16}{8}(15)^2 = 450 \text{ ft}$$