4.10 Antiderivatives

Definition: $F(x)$ is an antiderivative of $f(x)$ if $F'(x) = f(x)$.

Any two antiderivatives of $f(x)$ differ at most by a constant. So, if $F(x)$ is an antiderivative of $f(x)$, the most general antiderivative is

$F(x) + C$. C - arbitrary const.

Examples: Find the most general antiderivative of the given functions

1. $f(x) = 3x^2 + 5x - 2$ $\quad F(x) = x^3 + \frac{5}{2}x^2 - 2x + C$

2. $f(x) = 6\sqrt{x} - \sqrt[6]{x}$ $\quad F(x) = 6 \cdot \frac{2}{3} x^{3/2} - \frac{6}{7} x^{7/6} + C$

$= 4x\sqrt{x} - \frac{6}{7} x \sqrt[6]{x} + C$

3. $f(x) = 4e^x - \sec^2 x$ $\quad F(x) = 4e^x - \tan x + C$

4. $f(\theta) = \frac{\sin\theta}{\cos^2\theta}$ $\quad f(x) = \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta} = \tan\theta \sec\theta$

$F(x) = \sec\theta + C$

To evaluate the constant C one must be given conditions.

Ex Find $f(x)$

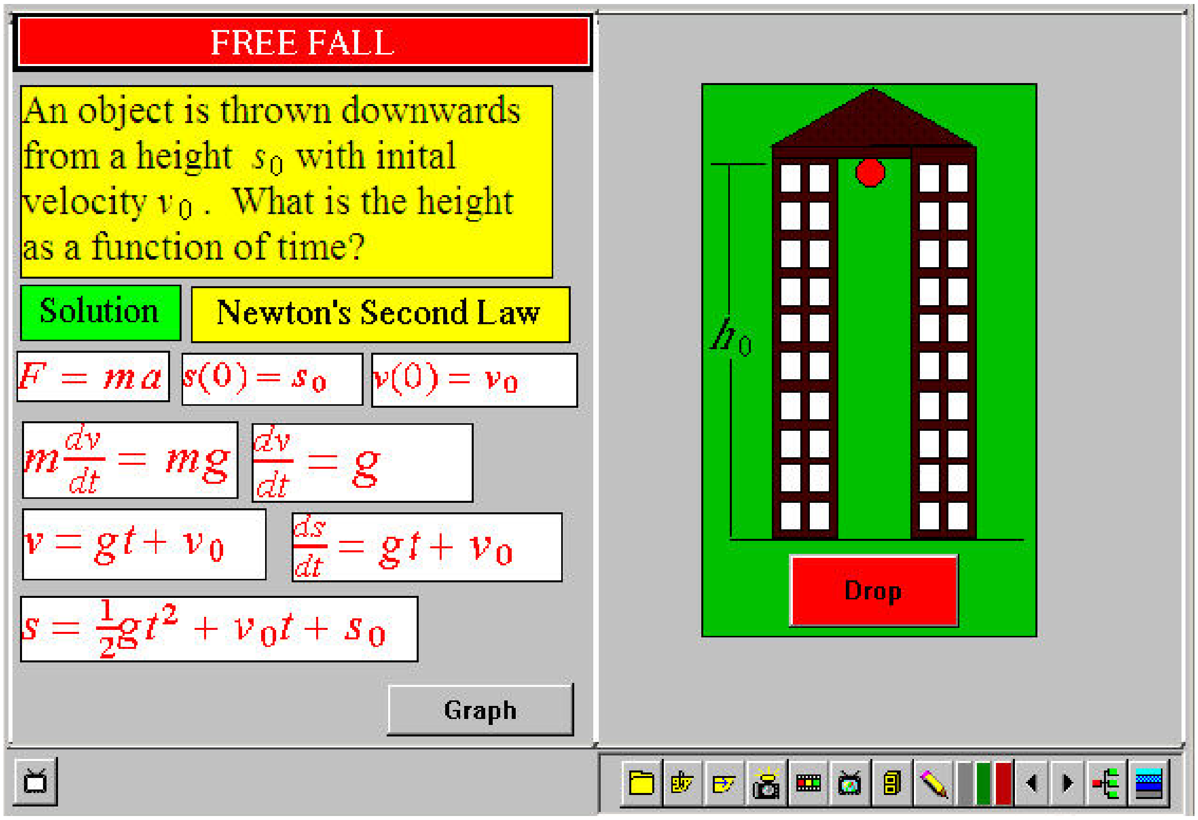
5. $f'(x) = 1 - 6x$ $\quad f(x) = x - 3x^2 + C$

$f(0) = 8$ $\quad f(0) = C = 8 \Rightarrow f(x) = x - 3x^2 + 8$

6. $f''(x) = 2 - 12x$ $\quad f'(x) = 2x - 6x^2 + C = 2x - 6x^2 + 2$

$f(0) = 3$ $\quad f(x) = x^2 - 2x^3 + 2x + D$

$f'(0) = 2$ $\quad = x^2 - 2x^3 + 2x + 3$



If one follows the convention that g is positive ($g = 9.8\ m/s^2$) but gravity points down, then the free fall equations become

$$\boxed{\begin{aligned} v &= -gt + v_0 \\ s &= -\tfrac{1}{2}gt^2 + v_0 t + s_0 \end{aligned}}$$

In USA units $g = 32\ ft/s^2$

Example: A stone was dropped off a cliff and hits the ground with a speed of 120 ft/s. What was the height of the cliff?

$v_0 = 0, \quad v = -gt + v_0$
$\qquad\qquad\;\; = -gt$

$s = -\frac{1}{2}gt^2 + s_0$

$-120 = -16t$

$t = \frac{120}{16} = \frac{15}{2}$

$0 = -\frac{1}{2}16\left(\frac{15}{2}\right)^2 + s_0$

$s_0 = \frac{16}{8}(15)^2 = 450 \text{ ft}$