2.3 Computing Limits

The Limit Theorem

Given $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = M$

Then a) $\lim_{x \to a} k f(x) = kL$

b) $\lim_{x \to a} (f+g)(x) = L + M$

c) $\lim_{x \to a} (f-g)(x) = L - M$

d) $\lim_{x \to a} (f \cdot g)(x) = L \cdot M$

e) $\lim_{x \to a} \left(\frac{f}{g}\right)(x) = \frac{L}{M}$ $\quad M \neq 0$

f) $\lim_{x \to a} f(g(x)) = f(M)$

Example: Compute and justify all steps

$$\lim_{x \to 1} \left(\frac{x^3 - 1}{x - 1}\right) = \lim_{x \to 1} \frac{\cancel{(x-1)}(x^2 + x + 1)}{\cancel{(x-1)}}$$

$$= \lim_{x \to 1} (x^2 + x + 1)$$

$$= \lim_{x \to 1} x^2 + \lim_{x \to 1} x + \lim_{x \to 1} (1)$$

$$= \left(\lim_{x \to 1} x\right)^2 + \lim_{x \to 1} x + \lim_{x \to 1} (1)$$

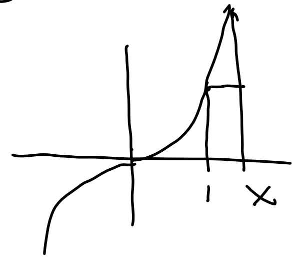
$$= 1 + 1 + 1 = 3$$

Note:
Let $y = f(x) = x^3$

x	$f(x)$
1	1
x	x^3

$m_{sec} = \frac{x^3 - 1}{x - 1}$

$m_{Tan} = \lim_{x \to 1} \frac{x^3 - 1}{x - 1}$



Example:
Compute.

Difference of Squares

$$\boxed{(a-b)(a+b) = a^2 - b^2}$$

$$\lim_{x \to 9} \left(\frac{\sqrt{x} - 3}{x - 9} \right)$$

$$= \lim_{x \to 9} \left(\frac{\sqrt{x} - 3}{x - 9} \right) \left(\frac{\sqrt{x} + 3}{\sqrt{x} + 3} \right)$$

$$= \lim_{x \to 9} \frac{\cancel{(x-9)}}{\cancel{(x-9)}(\sqrt{x}+3)} = \lim_{x \to 9} \frac{1}{\sqrt{x}+3}$$

$$= \frac{1}{\lim_{x \to 9} (\sqrt{x} + 3)} = \frac{1}{3+3} = \frac{1}{6}$$

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$$\lim_{x \to -4} \left(\frac{\frac{1}{4} + \frac{1}{x}}{4 + x} \right) = \lim_{x \to -4} \frac{1}{(4+x)} \left(\frac{1}{4} + \frac{1}{x} \right)$$

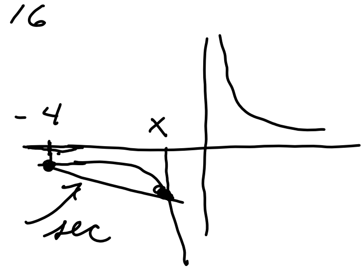
Common Denom.

$$= \lim_{x \to -4} \frac{1}{(4+x)} \left(\frac{x+4}{4x} \right)$$

$$= \lim_{x \to -4} \frac{1}{4x} = -\frac{1}{16}$$

Note:
Let $y = f(x) = \frac{1}{x}$

x	$f(x)$
-4	$-\frac{1}{4}$
x	$\frac{1}{x}$



$$m_{sec} = \frac{\frac{1}{x} + \frac{1}{4}}{x + 4}$$

$$m_{tan} = \lim_{x \to -4} \left(\frac{\frac{1}{4} + \frac{1}{x}}{4 + x} \right)$$

2.3b Computing Limits (Cont.)

Multiply by conjugate

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$$\lim_{x\to 0} \frac{x}{\sqrt{1+3x}-1} = \lim_{x\to 0}\left[\frac{x}{\sqrt{1+3x}-1}\right]\left[\frac{\sqrt{1+3x}+1}{\sqrt{1+3x}+1}\right]$$

$$= \lim_{x\to 0} \frac{x\left[\sqrt{1+3x}+1\right]}{(1+3x)-1}$$

$$= \lim_{x\to 0} \frac{x\left[\sqrt{1+3x}+1\right]}{3x}$$

$$= \lim_{x\to 0} \frac{\sqrt{1+3x}+1}{3} = \frac{2}{3}$$

When computing limits of fractions which contain radicals it often helps to rationalize (or antirationalize) by multiplying by the conjugate.