Review 1

Know definitions of inner products and be able to compute them

$$\mathbb{R}^n,\ \langle \mathbf{x}, \mathbf{y} \rangle = \sum_{i=1}^{n} x_i y_i,\ \ \mathbb{C}^n,\ \langle \mathbf{x}, \mathbf{y} \rangle = \sum_{i=1}^{n} x_i \overline{y_i}$$

$$\ell^2(\mathbb{Z}),\ \text{ Real case } \langle \mathbf{x}, \mathbf{y} \rangle = \sum_{i=\infty}^{\infty} x_i y_i,\ \text{ Complex case } \langle \mathbf{x}, \mathbf{y} \rangle = \sum_{i=\infty}^{\infty} x_i \overline{y_i}$$

$$L^2[0,1],\quad \text{Real case } \langle f(x), g(x) \rangle = \int_0^1 f(x) g(x)\, dx$$

$$L^2[0,1],\quad \text{Complex case } \langle f(x), g(x) \rangle = \int_0^1 f(x) \overline{g(x)}\, dx$$

Definition of norm $\|f\| = (\langle f, f \rangle)^{1/2}$, $\|\mathbf{x}\| = (\langle \mathbf{x}, \mathbf{x} \rangle)^{1/2}$, orthogonal vectors, orthogonal functions, orthogonal sequences.

Know definitions of periodic functions, Fourier coefficients/ Fourier Series. Be able to compute Fourier coefficients for simple cases (i.e. maybe an easy integration by parts).
Examples:

Complex case

$$c_n = \int_0^1 f(x) e^{-2\pi i n x} dx \text{ for } n \in \mathbb{Z},\ \sum_{n=-\infty}^{\infty} c_n e^{2\pi i n x}$$

Real case

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx,\ \ a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx \ n = 1, 2, 3 \cdots$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx \ n = 1, 2, 3 \ldots,\ \ a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx)$$

Know and be able to use Parseval's Identity, Weierstrass M-test, definition of uniformly continuous.

Euler's Formula $e^{ix} = \cos(x) + i \sin(x)$