Math 367 Homework 3

Directions: NEATLY write all solutions on your own paper. You may discuss the problems with each other but you must write them up independently.

We define the Fourier transform of a function $f(x)$ by

$$\hat{f}(w) = \mathcal{F}(f(x))(w) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i w x} dx$$

and the inverse Fourier transform by

$$f(x) = \mathcal{F}^{-1}(\hat{f}(w))(x) = \int_{-\infty}^{\infty} \hat{f}(w) e^{2\pi i w x} dw$$

1) Show $\mathcal{F}(e^{2\pi i k x} f(x))(w) = \hat{f}(w-k)$ and use it to find an expression for $\mathcal{F}(\cos(2\pi k x) f(x))(w)$. Hint: Euler's formula.

2) Show

a) $\mathcal{F}(x f(x))(w) = \frac{i}{2\pi} \frac{d}{dw}(\hat{f}(w))$

b) $\mathcal{F}(\mathcal{F}(f))(x) = f(-x)$

3) Given that $\mathcal{F}(e^{-ax^2})(w) = \sqrt{\frac{\pi}{a}} e^{-\frac{\pi^2 w^2}{a}}$ convince me why the convolution of two gaussians is a gaussian. Hint what is $\mathcal{F}(f * g)$?

4) It is easy to show that if $f(x)$ is a real valued function then $|\int_{-\infty}^{\infty} f(x) dx| \leq \int_{-\infty}^{\infty} |f(x)| dx$. The same is true for complex valued functions. Use this inequality to find a condition on $f(x)$ that makes its Fourier transform bounded.

5) Compute $\mathcal{F}(f(x))(w)$ for the function

$$f(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & x \leq 0 \end{cases}.$$