Math 367 Homework 2

Directions: NEATLY write all solutions on your own paper. Solutions should include details like integration by parts and reasons for convergence or divergence. For this assignment $f(x)$ are periodic functions so that $f : \mathbb{R} \to \mathbb{C}$. You may discuss the problems with each other but you must write them up independently.

1) Show that if $f(x)$ is 2π periodic and then $g(x) = f(px)$ is $P = \dfrac{2\pi}{p}$ periodic.
Now show that $\{\frac{1}{\sqrt{P}} e^{pinx}\}_{n\in\mathbb{Z}}$ is an orthonormal set on $[0, P]$.

2) **Using Euler's formula:** $e^{i\theta} = \cos(\theta) + i\sin(\theta)$ Use Euler's formula to show:
a) $\cos((n+m)\theta) = \cos(n\theta)\cos(m\theta) - \sin(n\theta)\sin(m\theta)$

b) $\cos(n\theta) + i\sin(n\theta) = (\cos(\theta) + i\sin(\theta))^n$.

c) If $z = x + iy$ then $z = re^{i\theta}$ and $\overline{z} = re^{-i\theta}$ with $r = \sqrt{x^2+y^2}$ and $\tan(\theta) = \dfrac{y}{x}$.

c) $|e^{ix}| = 1$ for all $x \in \mathbb{R}$.

3) If $f(x) = \sum_{n=-\infty}^{\infty} a_n e^{2\pi inx}$ and $g(x) = \sum_{m=-\infty}^{\infty} b_m e^{2\pi imx}$ find c_n in terms of a_n and b_n so $f(x)g(x) = \sum_{n=-\infty}^{\infty} c_n e^{2\pi inx}$.

4)Use the Weierstrass M-test to show that if $f(x) = \sum_{n=-\infty}^{\infty} a_n e^{2\pi inx}$ and $\sum_n |a_n| < \infty$ then $f(x)$ is a continuous function.