Math 367 Homework 1

Directions: NEATLY write all solutions on your own paper. Solutions should include details like integration by parts and reasons for convergence or divergence. You may discuss the problems with each other but you must write them up independently. All expansions are being done on the interval $[-\pi, \pi)$

1) Show by integrating that $\displaystyle\int_{-\pi}^{\pi} \cos^2(mx)dx = \pi$ for all $m \in \mathbb{N}$. Use the fact that $\int_0^T f(x)dx = \int_a^{a+T} f(x)dx$ for T periodic functions to conclude that $\displaystyle\int_{-\pi}^{\pi} \sin^2(mx)dx = \pi$ for all $m \in \mathbb{N}$.

2) Find the Fourier series for $f(x) = x^2$ for $-\pi \leq x \leq \pi$. Use the Fourier series for this function to prove $\displaystyle\sum \frac{1}{n^2} = \frac{\pi^2}{6}$. Hint: Find expansion then plug π into both sides.

3/4) **Convergence and Fourier series** Define the **saw tooth function** $f(x)$ to be

$$f(x) = \begin{cases} \frac{1}{2}(-\pi - x) & -\pi \leq x \leq 0 \\ \frac{1}{2}(\pi - x) & 0 < x \leq \pi \end{cases}.$$

We showed in class that the partial sums of the Fourier series are $s_N(x) = \displaystyle\sum_{n=1}^{N} \frac{\sin(nx)}{n}$. Show that

$$\lim_{N\to\infty} s_N(\frac{\pi}{N+1}) = \int_0^{\pi} \frac{\sin x}{x} dx$$

Next use the alternating series test and the Taylor series expansion of $\dfrac{sin(x)}{x}$ to show

$$1.85 < \int_0^{\pi} \frac{\sin x}{x} dx < 1.86.$$

Finally conclude that

$$\lim_{N\to\infty} |f(\frac{\pi}{N+1}) - s_N(\frac{\pi}{N+1})| \neq 0$$

5) **Differentiating Fourier series**. Again consider the saw tooth function above and its Fourier series. Show that the term by term derivative of the Fourier series does not converge to $f'(x)$.