MATH 335 **TEST 2p**

Name

Student number

1. (16 pts) True/False. Circle **T** or **F** (2 pts each). **Justify your answer. Examples or counter examples will suffice.** (2 pts each)

T F a. A mapping $T : \mathbb{R}^n \to \mathbb{R}^m$ is onto $\mathbb{R}^m$ if every vector $\mathbf{x}$ in $\mathbb{R}^n$ maps onto a vector in $\mathbb{R}^m$.

T F b. If A is an invertible matrix than the row reductions that reduce A to the identity matrix I also reduce A^{-1} to I.

T F c. If A is an $n \times n$ matrix whose columns are linearly independent then the columns span $\mathbb{R}^n$

T F d. If A and B are $n \times n$ matrices then $|A + B| = |A| + |B|$.

2. (10 pts) Let T be a linear transformation so that $T(x_1, x_2, x_3) = (x_1 + x_3, 4x_1, 2x_2)$. Determine the standard matrix for the linear transformation. Compute the inverse and use it to find $\mathbf{x}$ so $T(\mathbf{x}) = (-2, -4, 1)$.

3. (10 pts) Define **inverse matrices** and give an example of two matrices A and B so that $AB = I_2$ yet A and B are not inverses.

4. (10 pts) Find the inverse of the matrix and show it is correct by performing the multiplication and getting I_3.

$$\begin{bmatrix} 1 & 3 & -1 \\ 0 & 1 & 2 \\ -1 & 0 & 8 \end{bmatrix}$$

5. (10 pts) Prove that that if A is an $n \times n$ matrix and the columns of A span $\mathbb{R}^n$ then the columns are linearly independent.

6. (10 pts) Use Cramer's rule to solve system

$$5x_1 + 7x_2 = 3,\ 2x_1 + 4x_2 = 1.$$

7. (10 pts) The equation of a sphere of radius 1 in $\mathbb{R}^3$ is given by $x^2 + y^2 + z^2 = 1$. The map $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by the matrix $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ sends the sphere to an ellipsoid of the form $\dfrac{x^2}{4} + y^2 + \dfrac{z^2}{4} = 1$. Use determinants to find the volume of the ellipsoid.

8. (12 pts) a) Compute the determinant by reducing to row echelon form. Then check your answer using a cofactor expansion about the row or column of your choice.

$$\begin{bmatrix} 1 & 2 & 7 \\ 1 & 2 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

9. (12 pts) Let T be a transformation form $\mathbb{R}^3$ to $\mathbb{R}^3$ defined by $T(x_1, x_2, x_3) = (x_1 + x_2, x_2 + x_3, 2x_1 + 3x_2 + x_3)$. Find the standard matrix for this transformation and compute its determinant to determine if T is invertible.