MATH 335 Practice Test Name:

1. (12 pts) True/False. Circle **T** or **F** (1 pt each). Justify your answer. (2 pts each)

T F a. The column vectors of a matrix A are linearly independent if the the equation $A\mathbf{x} = \mathbf{0}$ has the trivial solution.

T F b. The equation $A\mathbf{x} = \mathbf{b}$ is homogeneous if the zero vector is a solution.

T F c. If the columns of a matrix A span $\mathbb{R}^m$ then $A\mathbf{x} = \mathbf{b}$ has a solution for each $\mathbf{b} \in \mathbb{R}^m$.

T F d. If $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^3$ one can always visualize the Span $\{\mathbf{u}, \mathbf{v}\}$ as a plane in $\mathbb{R}^3$.

2. (6 pts) Why is it not possible that the matrix $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 3 \end{bmatrix}$ is the reduced echelon form of the matrix $\begin{bmatrix} 2 & 0 & 2 \\ -1 & 0 & 2 \end{bmatrix}$

3. (12 pts) Define **consistent linear system** then determine for what value(s) h the matrix is the augmented matrix of a consistent linear system.

$$\begin{bmatrix} 3 & 6 & 9 \\ -1 & -2 & h \end{bmatrix}$$

4. (12 pts) Determine if $\mathbf{b}$ is in Span$\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$.

$$\mathbf{v_1} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix}, \mathbf{v_3} = \begin{bmatrix} -6 \\ 7 \\ 5 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$$

5. (12 pts) The matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & -7 & 8 \\ 6 & 7 & 8 & 7 \end{bmatrix}$ may be row reduced to $\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

If $\mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}$ and $A\mathbf{x} = \mathbf{b}$ has the solution $\begin{bmatrix} -13/4 \\ 43/8 \\ 0 \\ -15/8 \end{bmatrix}$, represent all the solutions.

6. (12 pts) Give the solution to the following system in parametric vector form.

$$\begin{aligned} x_1 \quad -3x_2 \quad -2x_3 &= -5 \\ x_2 \quad -x_3 &= 4 \\ -2x_1 \quad +3x_2 \quad +7x_3 &= -2 \end{aligned}$$

7. (12 pts) Determine which sets of vectors are linearly independent justify.

$a.\mathbf{v_1} = \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 1 \\ 2 \\ 8 \end{bmatrix}, \mathbf{v_3} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$ $\quad b.\mathbf{v_1} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \mathbf{v_3} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$ $\quad c.\ \mathbf{v_1} = \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 1 \\ 2 \\ 8 \end{bmatrix}$

8. (12 pts) Is the vector $\mathbf{b} = \begin{bmatrix} -4 \\ 9 \end{bmatrix}$ in the range of the linear transformation $T(\mathbf{x}) = A\mathbf{x}$ where $A = \begin{bmatrix} 1 & 0 & 3 \\ -2 & 1 & -3 \end{bmatrix}$? If so find $\mathbf{x}$ whose image under T is $\mathbf{b}$.

9. (12 pts) Let T be the linear transformation $T(x_1, x_2) = (2x_1 - x_2, x_1 + x_2, 0, x_1)$. State what spaces (i.e. $\mathbb{R}^n$) T map from and to and find the standard matrix for T. Next show that T is $1-1$.