The Behavior of a Multichannel Queueing System under Three Queue Disciplines

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Abstract

In this paper we investigate a multichannel channel queueing system where each channel consists of a limited capacity queue and server. Steady state system equations are generated for the three-channel case under each of three queue disciplines: ordered entry, semi-ordered entry and random shortest line. For given sums of channel capacities, we find channel capacity assignments, under each queue discipline, which minimize the probability of overflow of the system.

1 Introduction

Multichannel queueing models cover a wide variety of applications. Among these is the analysis of closed loop conveyor systems. Consider a conveyor system which feeds a station consisting of m parallel service channels, each channel performing the same service. We consider the case in which each of the m channels consists of a server with a limited capacity queue. This model is called a *multichannel queueing system*. Investigators have used the $M/M/m$

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queueing model to study this system. Although in recent years, this model has received only limited attention.

In this paper we build on the results of three papers authored by Elsayed and Lin, Elsayed, and Kodama and Fukuta respectively. We investigate a three-channel $M/M/3$ queueing system under the following assumptions and definitions:

1. The sum of the queue capacities is a fixed value S. The queue capacity of channels 1, 2, and 3 are $M, N,$ and L respectively. The queue capacity of each channel includes the server.

2. The service time for the server on channel i is exponentially distributed with rate μ_i.

3. The arrivals arrive from a common source and arrive according to a Poisson distribution with rate λ.

4. The traffic intensity is defined to be $\rho = \frac{\lambda}{\mu_1+\mu_2+\mu_3}$.

5. We consider three queue disciplines: ordered entry, semi-ordered entry, and random shortest line.

   Ordered entry: Arrivals select the channel with the lowest index that is not at full capacity.

   Semi-ordered entry: Arrivals select the channel with the shortest line. If two or more channels are tied, the arrival selects the channel with the lowest index among the channels that are tied.

   Random shortest line: Arrivals select the channel with the shortest line. If two or more channels are tied, the arrival selects a channel at random from the channels that are tied. (Similar to customers in a super market.)

6. The state of the system is (i, j, k) where i, j and k represent the number of arrivals at the first, second and third servers respectively including

the customer in service. The quantity $P_{i,j,k}$ represents the steady state probability that the system is in state (i,j,k).

We compare the three queue disciplines. Our comparison involves the probability of the system being idle, the probability of overflow, and the percentage of utilization of the three servers. Previous authors considered only the ordered entry and semi-ordered entry queue disciplines. We do not know of any previous papers that consider the random shortest line queue discipline.

2 Previous Results

Elsayed and Lin [2] studied the transient and steady state behaviour of ordered-entry multi-channel queueing systems under different operating conditions. They used a Runge-Kutta method to solve the difference-differential equations that define the system to determine the state probabilities at any time t. Steady state probabilities were calculated by taking the limit as $t \to \infty$. Elsayed and Lin present results on three conditions. They consider the effect of the servers' arrangement on the transient behaviour of the system, the effect of different queue space, and methodology for allocating the given waiting spaces among the servers. One of their conclusions is that the servers should be arranged in descending order in respect to their service rates to decrease the overflow probabilities.

Elsayed [1] considered the optimal allocation of storage space among the service channels for the ordered entry case by minimizing a cost function. The cost was a function of M, N, L and imposed a cost for:

- server idle times,
- space in the queues,
- waiting in the queues, and
- time server spends on an arrival.

For fixed values of S, λ, μ_1, μ_2, and μ_3, he solved the Kolmogorov birth-death steady-state equations for all possible feasible values of M, N, L (i.e. values such that $M + N + L = S$), to find an allocation of M, N, and K that minimizes the cost function. This was done for four different traffic intensities ($\rho = .2, .6, 1.0, 1.4$) and eight queue space allocations ($S = 1, \ldots, 8$). Elsayed considered both the finite and infinite source cases.

Kodama and Fukuta [3] introduced the queue discipline of semi-ordered entry. They compared the semi-ordered entry and ordered entry disciplines using the criteria of the probabilities of the system being idle and of overflow and the percentage of utilization of the servers . The did their comparison for the case of three channels, an arrival rate of $\lambda = 1.4$, services rates of $\mu_1 = 1.0, \mu_2 = .8$, and $\mu_3 = .6$, and queue capacities of (2,2,2), (3,2,1), and (1,2,3). Their conclusion was that "a queueing system with semi-ordered entry has more favourable properties than ordered entry."

3 Goals and Methodology

The purpose of this paper is three-fold:

- Introduce the queue discipline of random shortest line.
- Compare the three different queue disciplines for a variety of parameters.
- Investigate for given total queue capacity S, which allocation of spaces to the three queues is optimal for each queue discipline. This was done in [1] with the optimization criteria of minimizing cost. Our criteria is minimizing the probability of overflow.

For a given value of S, there are several ways to assign channel capacities, and the system states are undefined until these assignments are made. Without well defined system states the Kolmogorov birth-death steady-state equations cannot be solved. Thus, we use the method of *exhaustive enumeration* to find the queue capacities that minimize our criteria for each of the queue disciplines:

Step 1 Assign channel capacities M, N, L so that their sum is S.

Step 2 Generate the system of equations determined by the current queue discipline, the steady state relationship,

$$\text{rate into state } (i, j, k) = \text{rate out of state } (i, j, k),$$

and the boundary condition,

sum of steady state probabilities equals one.

Step 3 Solve for the steady state probabilities.

Step 4 If there is another assignment of channel capacities such that their sum is S, repeat steps 1-4.

Step 5 Identify the assignment of channel capacities that yields the minimum probability of overflow.

4 The model

For a given assignment of M, N, L each queue discipline generates a set of steady state equations. We let $\{P_{i,j,k}\}$ be the solution set to these equations where

The steady state equations for the ordered entry queue discipline, in the case of a finite source can be found in Elsayed [1]. From these, the equations for an infinite source are evident. The steady state equations which result from the semi-ordered entry queue can be derived from the Kolmogorov equations found in Kodama and Fukuta [3]. The steady state equations for the random shortest line queue discipline are not in the present literature. Because of several different cases to consider, there are 62 different equations. Thus, we do not list all of them here. We do, however, present several representative examples. Each equation may be characterized by the rate out of a particular state (the term on the left) equals the rate into that state.

The states $(0,0,k)$ with $M,N,L>1$:

$$\begin{aligned}
\lambda P_{0,0,0} &= \mu_3 P_{0,0,1} + \mu_2 P_{0,1,0} + \mu_1 P_{1,0,0} \\
(\lambda+\mu_3)P_{0,0,1} &= \frac{1}{3}\lambda P_{0,0,0} + \mu_3 P_{0,0,2} + \mu_2 P_{0,1,1} + \mu_1 P_{1,0,1} \\
(\lambda+\mu_3)P_{0,0,L} &= \mu_2 P_{0,1,L} + \mu_1 P_{1,0,L} \\
(\lambda+\mu_3)P_{0,0,k} &= \mu_3 P_{0,0,k+1} + \mu_2 P_{0,1,k} + \mu_1 P_{1,0,k} \ (L \geq 3 \ \ \wedge \ \ k = 2,\ldots,L-1)
\end{aligned}$$

The states (i,N,L) with $M,N,L>1$:

For $i = 1,\ldots,M-1$:

$$\begin{aligned}
&(i = N-1 \ \ \wedge \ \ i = L-1): \\
(\lambda+\mu_1+\mu_2+\mu_3)P_{i,N,L}\lambda P_{i-1,N,L} &= \frac{1}{2}\lambda(P_{i,N-1,L} + P_{i,N,L-1}) + \mu_1 P_{i+1,N,L} \\
&(i = N-1 \ \ \wedge \ \ i < L-1): \\
(\lambda+\mu_1+\mu_2+\mu_3)P_{i,N,L} &= \lambda P_{i-1,N,L} + \frac{1}{2}\lambda P_{i,N-1,L} + \mu_1 P_{i+1,N,L} \\
&(i = N-1 \ \ \wedge \ \ i > L-1): \\
(\lambda+\mu_1+\mu_2+\mu_3)P_{i,N,L} &= \lambda(P_{i-1,N,L} + P_{i,N,L-1}) + \frac{1}{2}\lambda P_{i,N-1,L} + \mu_1 P_{i+1,N,L} \\
&(i < N-1 \ \ \wedge \ \ i = L-1): \\
(\lambda+\mu_1+\mu_2+\mu_3)P_{i,N,L} &= \lambda P_{i-1,N,L} + \frac{1}{2}\lambda P_{i,N,L-1} + \mu_1 P_{i+1,N,L} \\
&(i < N-1 \ \ \wedge \ \ i < L-1): \\
(\lambda+\mu_1+\mu_2+\mu_3)P_{i,N,L} &= \lambda P_{i-1,N,L} + \mu_1 P_{i+1,N,L} \\
&(i < N-1 \ \ \wedge \ \ i > L-1): \\
(\lambda+\mu_1+\mu_2+\mu_3)P_{i,N,L} &= \lambda(P_{i-1,N,L} + P_{i,N,L-1}) + \mu_1 P_{i+1,N,L} \\
&(i > N-1 \ \ \wedge \ \ i = L-1): \\
(\lambda+\mu_1+\mu_2+\mu_3)P_{i,N,L} &= \lambda(P_{i-1,N,L} + P_{i,N-1,L}) + \frac{1}{2}\lambda P_{i,N,L-1} + \mu_1 P_{i+1,N,L} \\
&(i > N-1 \ \ \wedge \ \ i < L-1): \\
(\lambda+\mu_1+\mu_2+\mu_3)P_{i,N,L} &= \lambda(P_{i-1,N,L} + P_{i,N-1,L}) + \mu_1 P_{i+1,N,L} \\
&(i > N-1 \ \ \wedge \ \ i > L-1): \\
(\lambda+\mu_1+\mu_2+\mu_3)P_{i,N,L} &= \lambda(P_{i-1,N,L} + P_{i,N-1,L} + P_{i,N,L-1}) + \mu_1 P_{i+1,N,L} \\
(\mu_1+\mu_2+\mu_3)P_{M,N,L} &= \lambda(P_{M-1,N,L} + P_{M,N-1,L} + P_{M,N,L-1})
\end{aligned}$$

5 Results

A Maple program was written to find the steady state probabilities for each queue discipline for given values of: capacity sum, S, arrival rate λ, and service rates μ_1, μ_2, μ_3. The output was the values of the queue capacities M, N, and L that minimized the overflow for each queue discipline. However in some cases we printed out all values of the queue capacities in order to compare our results with [1] and [3]. Tables 1,2,and 3 in [3] are the most comparable to the work presented here. These tables do not contain optimal values of the queue capacities but contain probabilities of overflow, idleness, and server utilization for three values of M, N, and L for the six permutations of μ_1, μ_2, and μ_3. We were able to replicate tables 1 and 2 in [3] but not table 3. We believe table 3 contains some errors. This assertion is supported by Figure 3 in [2] which is a graph of the overflow probabilities for the same parameters as table 3 in [3]. Our results are very close to those in figure 3.

In the tables below, we include some cases contained in [1, 3] but our purpose is more to explore this queueing system with larger values of S. So, we include several cases not previously considered by other authors. We observe that in the case of random shortest line if M, N, L are optimal for particular values of λ and μ_1, μ_2, μ_3 and if we permute μ_1, μ_2, μ_3, then of course M, N, L will be permuted by the same permutation.

The notation in the tables is as follows:

oe: Ordered entry queue discipline

soe: Semi-ordered entry queue discipline

rsl: Random shortest line queue discipline

6 Conclusions

Kodama and Fukuta concluded that the semi-ordered entry discipline is superior to ordered entry and our results support that conclusion. It is interesting to note that the random shortest line discipline also is superior to ordered entry and is very close to semi-ordered entry even in semi-ordered entry's most favourable case, $\mu_1 > \mu_2 > \mu_3$, and is slightly superior in other cases. Thus, we conclude that if $\mu_1 > \mu_2 > \mu_3$, then semi-ordered entry is preferable but if the service rates are unknown or unreliable then semi-ordered entry and random shortest line perform equally well and are always superior to ordered entry.

7 REFERENCES

References

[1] E. A. Elsayed (1983), *Multichannel Queueing Systems With Ordered Entry and Finite Source*, Comput. & Ops. Res., Vol. **10**, p. 213.

[2] E. A. Elsayed and B. W. Lin (1980), *Transient Behaviour of Ordered-Entry Multichannel Queueing Systems*, Int. J. Prod. Res., Vol. **18**, p. 491.

[3] M. Kodama and J. Fukuta (1994), *On a Mutlichannel Queueing System with Semi-Ordered Entry*, J. Inf. & Optim. Sci., Vol. **15**, p. 65.

Table 1: Cases considered in [1], $S = 8, \lambda = 1.44$

Queue Discipline	Optimal Allocation M	N	L	Probability System Idle	Overflow	Percent Utilization Server 1	Server 2	Server 3
Service Rates: $\mu_1 = 1.2$ $\mu_2 = .8$ $\mu_3 = .4$								
oe	2	3	3	.144	.018	.725	.548	.263
soe	4	3	1	.161	.012	.647	.559	.494
rsl	5	2	1	.112	.014	.552	.610	.671
Service Rates: $\mu_1 = .8$ $\mu_2 = 1.2$ $\mu_3 = .4$								
oe	2	3	3	.096	.019	.834	.525	.286
soe	2	5	1	.133	.014	.729	.527	.511
rsl	2	5	1	.112	.014	.610	.552	.671
Service Rates: $\mu_1 = .8$ $\mu_2 = .4$ $\mu_3 = 1.2$								
oe	1	1	6	.104	.018	.643	.660	.530
soe	2	1	5	.101	.015	.737	.687	.461
rsl	2	1	5	.112	.014	.610	.671	.552
Service Rates: $\mu_1 = .4$ $\mu_2 = .8$ $\mu_3 = 1.2$								
oe	1	2	5	.068	.018	.783	.744	.421
soe	1	2	5	.082	.015	.783	.674	.472
rsl	1	2	5	.112	.014	.671	.610	.552
Service Rates: $\mu_1 = .4$ $\mu_2 = 1.2$ $\mu_3 = .8$								
oe	1	2	5	.084	.018	.783	.619	.447
soe	1	4	3	.094	.014	.783	.592	.496
rsl	1	5	2	.112	.014	.671	.552	.610
Service Rates: $\mu_1 = 1.2$ $\mu_2 = .4$ $\mu_3 = .8$								
oe	2	1	5	.134	.017	.725	.499	.431
soe	4	1	3	.136	.013	.653	.650	.472
rsl	5	1	2	.112	.014	.552	.671	.610

Table 2: Cases considered in [1], $S = 8, \lambda = 2.4$

Queue Discipline	Optimal Allocation M	N	L	Probability System Idle	Overflow	Percent Utilization Server 1	Server 2	Server 3
Service Rates: $\mu_1 = 1.2 \quad \mu_2 = .8 \quad \mu_3 = .4$								
oe	3	3	2	.017	.158	.933	.813	.629
soe	4	3	1	.025	.138	.885	.866	.781
rsl	4	3	1	.017	.142	.847	.890	.827
Service Rates: $\mu_1 = .8 \quad \mu_2 = 1.2 \quad \mu_3 = .4$								
oe	2	4	2	.014	.156	.923	.865	.623
soe	2	4	2	.018	.142	.895	.829	.874
rsl	3	4	1	.017	.142	.890	.847	.827
Service Rates: $\mu_1 = .8 \quad \mu_2 = .4 \quad \mu_3 = 1.2$								
oe	2	1	5	.013	.153	.923	.764	.823
soe	3	1	4	.015	.144	.930	.837	.812
rsl	3	1	4	.017	.142	.890	.827	.847
Service Rates: $\mu_1 = .4 \quad \mu_2 = .8 \quad \mu_3 = 1.2$								
oe	1	2	5	.011	.152	.857	.886	.820
soe	1	3	4	.013	.145	.857	.915	.814
rsl	1	3	4	.017	.142	.827	.890	.847
Service Rates: $\mu_1 = .4 \quad \mu_2 = 1.2 \quad \mu_3 = .8$								
oe	1	3	4	.012	.154	.857	.890	.775
soe	1	4	3	.015	.141	.857	.868	.845
rsl	1	4	3	.017	.142	.827	.847	.890
Service Rates: $\mu_1 = 1.2 \quad \mu_2 = .4 \quad \mu_3 = .8$								
oe	3	1	4	.017	.158	.933	.682	.785
soe	4	1	3	.021	.140	.887	.823	.839
rsl	4	1	3	.017	.142	.847	.827	.890

Table 3: Cases considered in [1], $S = 8, \lambda = 3.36$

Queue Discipline	Optimal Allocation M	N	L	Probability System Idle	Overflow	Percent Utilization Server 1	Server 2	Server 3
Service Rates: $\mu_1 = 1.2$ $\mu_2 = .8$ $\mu_3 = .4$								
oe	3	3	2	.002	.329	.970	.934	.859
soe	3	3	2	.003	.319	.950	.955	.963
rsl	3	3	2	.002	.322	.932	.962	.976
Service Rates: $\mu_1 = .8$ $\mu_2 = 1.2$ $\mu_3 = .4$								
oe	2	4	2	.002	.328	.956	.957	.857
soe	2	4	2	.002	.321	.950	.948	.962
rsl	3	3	2	.002	.322	.962	.932	.976
Service Rates: $\mu_1 = .8$ $\mu_2 = .4$ $\mu_3 = 1.2$								
oe	2	2	4	.001	.329	.956	.969	.918
soe	2	2	4	.002	.323	.950	.974	.938
rsl	3	2	3	.002	.322	.962	.976	.932
Service Rates: $\mu_1 = .4$ $\mu_2 = .8$ $\mu_3 = 1.2$								
oe	1	2	5	.002	.329	.894	.939	.955
soe	1	3	4	.002	.324	.894	.974	.946
rsl	2	3	3	.002	.322	.976	.962	.932
Service Rates: $\mu_1 = .4$ $\mu_2 = 1.2$ $\mu_3 = .8$								
oe	1	3	4	.002	.329	.894	.953	.941
soe	2	3	3	.002	.322	.984	.940	.946
rsl	2	3	3	.002	.322	.976	.932	.962
Service Rates: $\mu_1 = 1.2$ $\mu_2 = .4$ $\mu_3 = .8$								
oe	3	2	3	.002	.330	.970	.948	.885
soe	3	2	3	.002	.320	.950	.974	.944
rsl	3	2	3	.002	.322	.932	.976	.962

Table 4: Cases considered in [2] and [3], $S = 6, \lambda = 1.4$

Queue Discipline	Optimal Allocation			Probability		Percent Utilization		
	M	N	L	System Idle	Overflow	Server 1	Server 2	Server 3
Service Rates: $\mu_1 = 1.0 \quad \mu_2 = .8 \quad \mu_3 = .6$								
oe	2	2	2	.137	.043	.771	.512	.265
soe	2	2	2	.169	.032	.660	.542	.437
rsl	3	2	1	.145	.034	.546	.580	.570
Service Rates: $\mu_1 = .8 \quad \mu_2 = 1.0 \quad \mu_3 = .6$								
oe	1	2	3	.143	.040	.636	.590	.408
soe	2	2	2	.153	.033	.715	.513	.447
rsl	2	3	1	.145	.034	.580	.546	.570
Service Rates: $\mu_1 = .8 \quad \mu_2 = .6 \quad \mu_3 = 1.0$								
oe	1	1	4	.136	.040	.636	.557	.501
soe	2	1	3	.136	.036	.719	.584	.425
rsl	2	1	3	.145	.034	.580	.570	.546
Service Rates: $\mu_1 = .6 \quad \mu_2 = .8 \quad \mu_3 = 1.0$								
oe	1	1	4	.123	.041	.700	.516	.510
soe	1	2	3	.124	.036	.700	.623	.431
rsl	1	2	3	.145	.034	.570	.580	.546
Service Rates: $\mu_1 = .6 \quad \mu_2 = 1.0 \quad \mu_3 = .8$								
oe	1	2	3	.122	.040	.700	.625	.373
soe	1	3	2	.133	.035	.700	.582	.438
rsl	1	3	2	.145	.034	.570	.546	.580
Service Rates: $\mu_1 = 1.0 \quad \mu_2 = .6 \quad \mu_3 = .8$								
oe	2	1	3	.136	.042	.771	.435	.387
soe	3	1	2	.159	.033	.676	.564	.424
rsl	3	1	2	.145	.034	.546	.570	.580

Table 5: $S = 10, \lambda = 2.4$

Queue Discipline	Optimal Allocation M	N	L	Probability System Idle	Overflow	Percent Utilization Server 1	Server 2	Server 3
Service Rates: $\mu_1 = 1.2 \quad \mu_2 = .8 \quad \mu_3 = .4$								
oe	3	4	3	.011	.128	.933	.871	.688
soe	5	3	2	.017	.109	.899	.878	.892
rsl	5	3	2	.011	.113	.863	.896	.939
Service Rates: $\mu_1 = .8 \quad \mu_2 = 1.2 \quad \mu_3 = .4$								
oe	2	5	3	.009	.128	.923	.902	.683
soe	3	5	2	.014	.112	.932	.856	.897
rsl	3	5	2	.011	.113	.896	.863	.939
Service Rates: $\mu_1 = .8 \quad \mu_2 = .4 \quad \mu_3 = 1.2$								
oe	2	2	6	.006	.126	.923	.929	.824
soe	3	2	5	.009	.115	.935	.944	.832
rsl	3	2	5	.011	.113	.896	.939	.863
Service Rates: $\mu_1 = .4 \quad \mu_2 = .8 \quad \mu_3 = 1.2$								
oe	1	2	7	.008	.126	.857	.886	.872
soe	2	3	5	.007	.116	.965	.917	.835
rsl	2	3	5	.011	.113	.939	.896	.863
Service Rates: $\mu_1 = .4 \quad \mu_2 = 1.2 \quad \mu_3 = .8$								
oe	1	4	5	.008	.128	.857	.936	.784
soe	2	5	3	.008	.114	.964	.881	.855
rsl	2	5	3	.011	.113	.939	.863	.896
Service Rates: $\mu_1 = 1.2 \quad \mu_2 = .4 \quad \mu_3 = .8$								
oe	3	2	5	.010	.126	.933	.869	.786
soe	5	2	3	.014	.111	.901	.935	.848
rsl	5	2	3	.011	.113	.863	.939	.896

Table 6: $S = 12, \lambda = 2.4$

Queue Discipline	Optimal Allocation M	N	L	Probability System Idle	Overflow	Percent Utilization Server 1	Server 2	Server 3
Service Rates: $\mu_1 = 1.2$ $\mu_2 = .8$ $\mu_3 = .4$								
oe	3	4	5	.007	.109	.933	.871	.805
soe	6	4	2	.014	.090	.915	.904	.908
rsl	6	4	2	.008	.093	.885	.921	.946
Service Rates: $\mu_1 = .8$ $\mu_2 = 1.2$ $\mu_3 = .4$								
oe	2	5	5	.006	.108	.923	.902	.800
soe	4	6	2	.011	.092	.950	.878	.912
rsl	4	6	2	.008	.093	.921	.885	.946
Service Rates: $\mu_1 = .8$ $\mu_2 = .4$ $\mu_3 = 1.2$								
oe	2	2	8	.004	.105	.923	.929	.865
soe	4	2	6	.007	.095	.952	.951	.858
rsl	4	2	6	.008	.093	.921	.946	.885
Service Rates: $\mu_1 = .4$ $\mu_2 = .8$ $\mu_3 = 1.2$								
oe	1	3	8	.004	.106	.857	.953	.866
soe	2	4	6	.005	.095	.967	.939	.861
rsl	2	4	6	.008	.093	.946	.921	.885
Service Rates: $\mu_1 = .4$ $\mu_2 = 1.2$ $\mu_3 = .8$								
oe	1	4	7	.006	.108	.857	.936	.843
soe	2	6	4	.007	.093	.967	.900	.887
rsl	2	6	4	.008	.093	.946	.885	.921
Service Rates: $\mu_1 = 1.2$ $\mu_2 = .4$ $\mu_3 = .8$								
oe	3	2	7	.007	.107	.933	.869	.845
soe	6	2	4	.011	.091	.916	.942	.880
rsl	6	2	4	.008	.093	.885	.946	.921

Table 7: $S = 14, \lambda = 2.8$

Queue Discipline	Optimal Allocation M	N	L	Probability System Idle	Overflow	Percent Utilization Server 1	Server 2	Server 3
Service Rates: $\mu_1 = 1.2 \quad \mu_2 = .8 \quad \mu_3 = .4$								
oe	4	5	5	.001	.180	.980	.949	.899
soe	7	4	3	.002	.167	.972	.969	.979
rsl	7	5	2	.002	.168	.964	.978	.974
Service Rates: $\mu_1 = .8 \quad \mu_2 = 1.2 \quad \mu_3 = .4$								
oe	3	6	5	.001	.180	.983	.960	.898
soe	4	7	3	.002	.168	.983	.959	.980
rsl	5	7	2	.002	.168	.978	.964	.974
Service Rates: $\mu_1 = .8 \quad \mu_2 = .4 \quad \mu_3 = 1.2$								
oe	3	2	9	.001	.177	.983	.946	.949
soe	4	2	8	.001	.169	.983	.975	.958
rsl	5	2	7	.002	.168	.978	.974	.964
Service Rates: $\mu_1 = .4 \quad \mu_2 = .8 \quad \mu_3 = 1.2$								
oe	2	3	9	.000	.177	.982	.965	.950
soe	2	5	7	.001	.170	.980	.983	.955
rsl	2	5	7	.002	.168	.974	.978	.964
Service Rates: $\mu_1 = .4 \quad \mu_2 = 1.2 \quad \mu_3 = .8$								
oe	2	5	7	.000	.179	.982	.976	.919
soe	2	7	5	.001	.168	.980	.969	.968
rsl	2	7	5	.002	.168	.974	.964	.978
Service Rates: $\mu_1 = 1.2 \quad \mu_2 = .4 \quad \mu_3 = .8$								
oe	4	3	7	.001	.179	.980	.962	.920
soe	7	2	5	.002	.167	.974	.973	.967
rsl	7	2	5	.002	.168	.964	.974	.978

Table 8: $S = 16, \lambda = 2.8$

Queue Discipline	Optimal Allocation M	N	L	Probability System Idle	Overflow	Percent Utilization Server 1	Server 2	Server 3
Service Rates: $\mu_1 = 1.2$ $\mu_2 = .8$ $\mu_3 = .4$								
oe	4	6	6	.001	.172	.980	.967	.918
soe	8	5	3	.002	.160	.979	.979	.985
rsl	8	5	3	.001	.161	.971	.982	.992
Service Rates: $\mu_1 = .8$ $\mu_2 = 1.2$ $\mu_3 = .4$								
oe	3	7	6	.000	.171	.983	.973	.916
soe	5	8	3	.001	.161	.989	.970	.985
rsl	5	8	3	.001	.161	.982	.971	.992
Service Rates: $\mu_1 = .8$ $\mu_2 = .4$ $\mu_3 = 1.2$								
oe	3	3	10	.000	.169	.983	.985	.955
soe	5	3	8	.001	.162	.990	.993	.965
rsl	5	3	8	.001	.161	.982	.992	.971
Service Rates: $\mu_1 = .4$ $\mu_2 = .8$ $\mu_3 = 1.2$								
oe	2	4	10	.000	.169	.982	.987	.954
soe	2	5	9	.001	.162	.981	.987	.969
rsl	3	5	8	.001	.161	.992	.982	.971
Service Rates: $\mu_1 = .4$ $\mu_2 = 1.2$ $\mu_3 = .8$								
oe	2	5	9	.000	.170	.982	.976	.949
soe	3	8	5	.001	.161	.995	.976	.974
rsl	3	8	5	.001	.161	.992	.971	.982
Service Rates: $\mu_1 = 1.2$ $\mu_2 = .4$ $\mu_3 = .8$								
oe	4	3	9	.001	.171	.980	.962	.950
soe	8	3	5	.001	.160	.980	.991	.973
rsl	8	3	5	.001	.161	.971	.992	.982

Table 9: $S = 20, \lambda = 2.8$

Queue Discipline	Optimal Allocation M	N	L	Probability System Idle	Overflow	Percent Utilization Server 1	Server 2	Server 3
Service Rates: $\mu_1 = 1.2$ $\mu_2 = .8$ $\mu_3 = .4$								
oe	5	7	8	.000	.161	.992	.976	.944
soe	10	7	3	.001	.152	.989	.990	.991
rsl	11	6	3	.000	.153	.985	.991	.994
Service Rates: $\mu_1 = .8$ $\mu_2 = 1.2$ $\mu_3 = .4$								
oe	4	8	8	.000	.161	.995	.980	.944
soe	6	11	3	.001	.152	.995	.984	.991
rsl	6	11	3	.000	.153	.991	.985	.994
Service Rates: $\mu_1 = .8$ $\mu_2 = .4$ $\mu_3 = 1.2$								
oe	4	3	13	.000	.158	.995	.983	.973
soe	6	3	11	.000	.153	.995	.995	.982
rsl	6	3	11	.000	.153	.991	.994	.985
Service Rates: $\mu_1 = .4$ $\mu_2 = .8$ $\mu_3 = 1.2$								
oe	2	4	14	.000	.158	.982	.987	.979
soe	3	6	11	.000	.153	.996	.993	.982
rsl	3	6	11	.000	.153	.994	.991	.985
Service Rates: $\mu_1 = .4$ $\mu_2 = 1.2$ $\mu_3 = .8$								
oe	2	6	12	.000	.160	.982	.987	.969
soe	3	10	7	.000	.153	.996	.987	.988
rsl	3	11	6	.000	.153	.994	.985	.991
Service Rates: $\mu_1 = 1.2$ $\mu_2 = .4$ $\mu_3 = .8$								
oe	5	4	11	.000	.160	.992	.983	.961
soe	10	3	7	.001	.152	.989	.994	.987
rsl	11	3	6	.000	.153	.985	.994	.991