1. What is the last digit of 3^{2013}?

   A. 1 B. 3 C. 5 D. 7 E. 9

2. Two numbers x and y are such that $x + y = 20$ and $\frac{1}{x} + \frac{1}{y} = \frac{1}{2}$. What is the value of $x^2y + xy^2$?

   A. 80 B. 200 C. 400 D. 640 E. 800

3. In the diagram (not drawn to scale), $\angle ABE = 10°$; $\angle EBC = 70°$; $\angle ACD = 50°$; $\angle DCB = 20°$; $\angle DEF = \alpha$. Which of the following is equal to $\tan\alpha$?

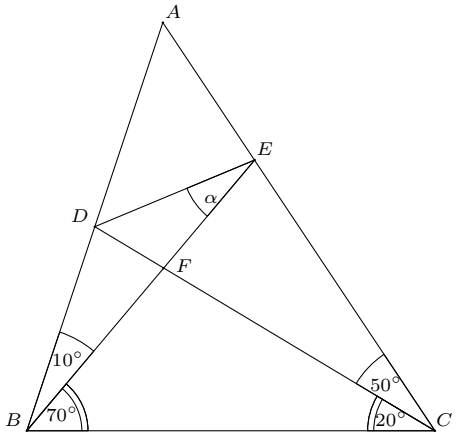
   A. $\frac{\tan 10° \tan 20°}{\tan 50°}$

   B. $\frac{\tan 10° \tan 20°}{\tan 70°}$

   C. $\frac{\tan 10° \tan 50°}{\tan 70°}$

   D. $\frac{\tan 20° \tan 50°}{\tan 70°}$

   E. $\frac{\tan 10° \tan 70°}{\tan 50°}$



4. At present Mary is 6 times as old as her daughter Susan. Four years from now she will be only four times as old as Susan. What is the sum of their ages at present?

   A. 44 B. 42 C. 49 D. 35 E. 56

5. Let f be an even function. Find the slope of the line passing through the points $(2, f(2))$ and $(-2, f(-2))$ on the graph of $y = f(x)$?

   A. 1 B. $-f(2)$ C. $\frac{1}{2}f(2)$ D. $2f(2)$ E. 0

6. Given the two circles defined by the equations $x^2 - 6x + y^2 + 8y = 12$ and $x^2 + y^2 = 4y$, find the algebraic equation of the line connecting their centers.

   A. $x + 2y = 4$
   B. $2x + y = 2$
   C. $4x + y = 2$
   D. $2x + y = -4$
   E. none of these

7. Let r and s be the roots of the equation $x^2 + bx + c = 0$. What is the value of $\frac{r^3+s^3}{r^2+s^2-rs}$ in terms of b and c?

   A. b B. c C. $b + c$ D. bc E. $-b$

8. Find the area of a regular hexagon whose vertices lie on a circle of radius 4.

   A. 16π B. $2\pi\sqrt{3}$ C. $16\sqrt{3}$ D. $24\sqrt{3}$ E. none of these

9. How many positive integers less than 700 are NOT divisible by 8?

   A. 612 B. 620 C. 634 D. 637 E. 654

10. A bug lives on a corner of a cube and is allowed to travel only on the edges of the cube. In how many ways can the bug visit each of the other seven corners once and only once, returning to its home corner only at the end of the trip?

    A. 6 B. 8 C. 12 D. 24 E. 27

11. Which of the following is a representation of $\sec^5\theta\cot\theta - \sec^3\theta\cot\theta$?

    A. $\dfrac{\cos\theta}{\sin^2\theta}$ B. $\dfrac{\sin\theta}{\cos^2\theta}$ C. $\dfrac{\sin\theta}{\cos^4\theta}$ D. $\sec^2\theta\cot\theta$ E. $\dfrac{\cos^4\theta}{\sin\theta}$

12. Consider the sequence recursively defined by $a_1 = 1$, $a_2 = 2$, and for $n > 2$, $a_n = \frac{1+a_{n-1}}{a_{n-2}}$. What is the 10^{th} term of the sequence?

    A. 1 B. 2 C. 3 D. 4 E. none of the above

13. A function $f(x)$ satisfies the equation $f\big(4x(x+1)\big) = 2\big(f(x)\big)^2 - 1$. If $f(0) > 0$, $f(-1) > 0$ and $f\left(-\frac{1}{2}\right) < 0$, then what is the value of $f\big(-\frac{1}{2}\big)$?

A. -3 B. -1 C. $-\frac{1}{2}$ D. 0 E. 1

14. Let x and y be positive integers with $x > y$. If $\dfrac{1}{x+y} + \dfrac{1}{x-y} = \dfrac{1}{3}$, find $x^2 + y^2$.

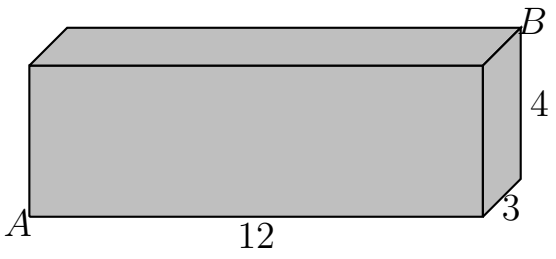
A. 52 B. 58 C. 65 D. 73 E. 80

15. If $\sin x + \cos x = \dfrac{1}{2}$, what is the value of $\sin^3 x + \cos^3 x$?

A. $\dfrac{1}{8}$ B. $\dfrac{5}{16}$ C. $\dfrac{3}{8}$ D. $\dfrac{5}{8}$ E. $\dfrac{11}{16}$

16. Find the distance between point A and point B in the rectangular box below.

A. 13
B. 19
C. $\sqrt{151}$
D. $4\sqrt{10} + 3$
E. $4\sqrt{10} + \sqrt{3}$



17. There are n silver coins and 6 gold coins in a box. Determine n given that the probability of randomly picking one gold coin is 0.25.

A. 2 B. 3 C. 6 D. 12 E. 18

18. Find the sum: $\frac{1}{2} + \frac{1}{3} - \frac{1}{4} - \frac{1}{9} + \frac{1}{8} + \frac{1}{27} - \frac{1}{16} - \frac{1}{81} + \cdots$

A. $\frac{1}{6}$ B. $\frac{3}{2}$ C. $\frac{5}{6}$ D. $\frac{5}{12}$ E. $\frac{7}{12}$

19. What is the probability that the sum of two different randomly picked numbers from $1, 2, 3, 4, 5$ is even?

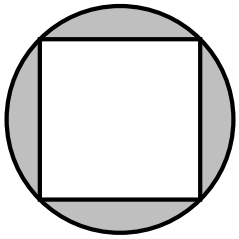
A. 0.167 B. 0.4 C. 0.5 D. 0.6 E. 0.833

20. There are three different colors of balls in a box: red, blue, and white. If one ball is randomly chosen, the chance that it is red is 0.3, and the chance that it is blue is 0.3. What is the probability of picking a white one?

A. 0.3 B. $\frac{1}{3}$ C. 0.4 D. 0.5 E. $\frac{2}{3}$

21. A square is inscribed in a circle of radius 10. If the gray regions of the circle are folded over onto the square, what is the area of the portion of the square that will remain uncovered?

A. $100\pi - 100$
B. $100\pi - 200$
C. $200 - 100\pi$
D. $400 - 100\pi$
E. none of these



22. You have 200 coins. You give them to your friends in such a way that each friend gets at least one coin and no two friends get the same number of coins. What is the largest number of friends that you could have?

A. 16 B. 17 C. 19 D. 20 E. 23

23. Find all real and complex solutions: $x^6 - 1 = 0$

A. $1, -1$
B. $1, -1, i, -i$
C. $1, -1, \frac{\sqrt{3}}{2} \pm \frac{1}{2}i, -\frac{\sqrt{3}}{2} \pm \frac{1}{2}i$
D. $1, -1, \frac{1}{2} \pm \frac{\sqrt{3}}{2}i, -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$
E. $1, -1, \frac{\sqrt{2}}{2} \pm \frac{\sqrt{2}}{2}i, -\frac{\sqrt{2}}{2} \pm \frac{\sqrt{2}}{2}i$

24. If the real zeros of the polynomial $x^5 + 15x^4 + 2x^3 + 22x^2 - 115x + 75$ are 1 and -15, what are the complex zeros?

A. $-1 \pm 2i$ B. $-2 \pm 2i$ C. $-1 \pm 2\sqrt{2}i$ D. $-2 \pm 2\sqrt{2}i$ E. $-2 \pm 4i$

25. A panel of judges for a fashion show consists of 4 women and 3 men. How many ways can the judges be seated in a row of seven chairs if all of the men must sit together?

A. 144 B. 2 C. 288 D. 256 E. 720

26. In scheduling final exams for summer school at Central High, six different tests have to be given to seven students. The table below shows the exams that each of the students must take. What is the minimum number of time slots that will be needed to schedule the six exams?

Exams	**Amy**	**Ben**	**Charles**	**Debra**	**Ed**	**Frank**	**Georgia**
Math	X		X		X		X
Art		X		X		X	
Science	X	X					X
History			X			X	
French					X	X	
Reading	X	X		X	X		X

A. 2 B. 3 C. 4 D. 5 E. 6

27. Suppose the complex numbers x and y satisfy $x^3+y^3 = 25i$ and $x+y = 5$ and write $xy = a + bi$ for some real numbers a and b. What is the value of $a + b$?

A. -20 B. -10 C. $-\frac{10}{3}$ D. -1 E. $\frac{20}{3}$

28. Find the area of a rectangle inscribed in the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ if the edges of the rectangle cross the foci.

A. $\sqrt{7}$ B. 28 C. $\frac{3}{2}\sqrt{7}$ D. $9\sqrt{7}$ E. $\frac{9}{2}\sqrt{7}$

29. Factor completely: $x^{64} - 1$

A. $(x+1)^8(x-1)^8$
B. $(x+1)^{32}(x-1)^{32}$
C. $(x^{32}+1)(x^{16}+1)(x^8+1)(x^4+1)(x^2+1)(x+1)(x-1)$
D. $(x^{32}+1)(x^{32}-1)$
E. $(x^{32}+x^{16}+x^8+x^4+x^2+x+1)(x^{32}-x^{16}-x^8-x^4-x^2-x-1)$

30. You have some coins in your pocket. You discover that you cannot choose a set of these coins to get a total of exactly \$1.00. What is the largest possible total value of the coins in your pocket? Coins can be worth \$0.01, \$0.05, \$0.10, \$0.25.

A. \$0.99 B. \$1.09 C. \$1.19 D. \$1.25 E. \$1.31