**UNCW High School Mathematics Contest** **Spring 2011**

1. A teacher has some M&M's that he would like to share with his students. He wants to give 4 to each student, but he will be short 2. He decides to give only 3 to each of his students, which will leave him with 1. How many students does the teacher have?

   A. 3 B. 4 C. 5 D. 6 E. 8

2. If $\frac{b}{a} = 2$ and $\frac{c}{b} = 3$, find the value of $\frac{a+b}{b+c}$.

   A. $\frac{1}{3}$ B. $\frac{3}{8}$ C. $\frac{3}{5}$ D. $\frac{2}{3}$ E. $\frac{3}{4}$

3. The shortest distance between the point $(6, -1)$ and the line $3x - y = 4$ is:

   A. $3\sqrt{6}$ B. 45 C. $\frac{\sqrt{5465}}{2}$ D. $\frac{3\sqrt{10}}{2}$ E. None of these

4. Find the sum of the infinite series for $x > 1$.

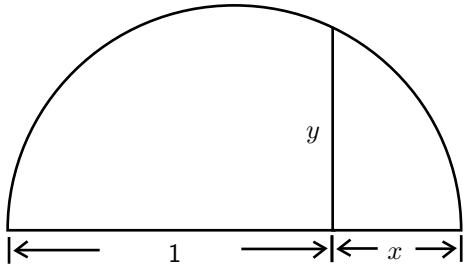
$$\frac{1}{x} + \frac{2}{x^2} + \frac{3}{x^3} + \cdots + \frac{n}{x^n} + \ldots$$

   A. $\frac{x}{x-1}$ B. $x(x-1)$ C. $\frac{x}{(x-1)^2}$ D. $\frac{x^2}{(x-1)^2}$ E. $x(x-1)^2$

5. The equation of the reflection of the line $2x + 3y = 6$ in the mirror placed at the line $x = 3$ is:

   A. $-x + 2y = 1$
   B. $3y - 2x = 6$
   C. $x - 2y = 1$
   D. $2x - 3y = 6$
   E. $x + 2y = 1$

6. Find the length $y$ of the vertical segment in the figure, if the horizontal segment is the diameter of the semicircle.




A. 1 B. $\sqrt{1+x^2}$ C. $\sqrt{x}$ D. $\sqrt{1-x^2}$ E. $x$

7. Given that $y = (f \circ g)(x) = 6x^2 - 23$ and $g(x) = x^2 - 4$, $x \geq 0$. Find $f(0)$.

A. $-23$ B. $-4$ C. 1 D. 73 E. 524

8. The altitude of a right circular cone is 15 and the radius of its base is 8. A cylindrical hole of radius 2 is drilled through the cone, with its axis along the axis of the cone. What is the volume of the solid that remains?

A. $250\pi$ B. $260\pi$ C. $270\pi$ D. $280\pi$ E. None of these

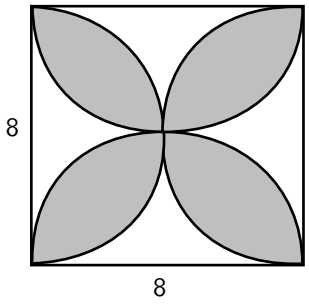
9. Let $f(x)$ be a function such that, for every real number $x$, $f(-x) + 2f(x) = \cos x$. What is the value of $f\left(\frac{\pi}{4}\right)$?

A. $\sqrt{2}$ B. $\frac{1}{2}$ C. $\frac{\sqrt{3}}{2}$ D. $\frac{\sqrt{3}}{4}$ E. $\frac{\sqrt{2}}{6}$

10. Simplify $\frac{\sin x}{1-\cos x} - \frac{\sin x}{1+\cos x}$.

A. $2\tan x$
B. $\cos x - \sin x$
C. $2\cot x$
D. $\cos x + \sin x$
E. None of these

11. The figure is a square with four intersecting semicircles forming the "flower". Find the probability that a randomly selected point in the square falls in the shaded region.




A. $\frac{1}{2}$ B. $\frac{9}{16}$ C. $\frac{1}{4}(\pi - 1)$ D. $1 - \frac{\pi}{4}$ E. $\frac{\pi}{2} - 1$

12. Let $g(x) = \frac{a^x + a^{-x}}{a^x - a^{-x}}$, where $a > 0$ and $a \neq 1$. Given $g(p) = 2$, determine $g(2p)$.

A. 2 B. 3 C. $\sqrt{2}$ D. $\frac{5}{4}$ E. $\frac{8}{7}$

13. A vertical line divides the triangle with vertices $(0, 0), (1, 1)$ and $(9, 1)$ in the $xy$-plane into 2 regions of equal area. If the equation of the line is $x = k$, then $k$ is:

A. 2.5 B. 3.0 C. 3.5 D. 4.0 E. 4.5

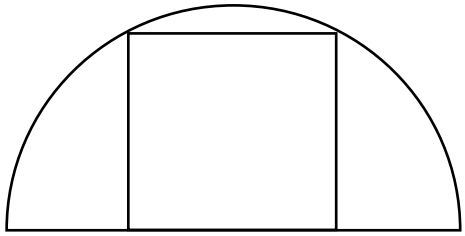
14. What is the value of $81^{-(2^{-2})}$?

A. $\frac{1}{9}$ B. 43046721 C. $\frac{1}{3}$ D. 6561 E. $\frac{\sqrt{3}}{3}$

15. What is the value of $(\log_4 8) \cdot (\log_{27} 9)$?

A. $\frac{4}{9}$ B. 1 C. $\frac{2}{3}$ D. $\frac{3}{2}$ E. 2

16. Find the ratio of the area of the semicircle to the area of the inscribed square.



A. $\pi$ B. $2\sqrt{2}$ C. $\frac{\pi}{2}$ D. 2 E. $\frac{5\pi}{8}$

17. When three people are weighed two at a time, their average weights are 140, 150, and 170 pounds. How much does the lightest person weigh?

A. 100 B. 120 C. 150 D. 160 E. 170

18. Five distinct integers are randomly selected from 1 to 10. Find the probability that the largest number selected is 7.

A. $\frac{5}{84}$ B. $\frac{1}{14}$ C. $\frac{1}{12}$ D. $\frac{5}{36}$ E. $\frac{7}{43}$

19. In a group of 40 students, there are 33 who take algebra class, 29 who take geometry class and 23 who take calculus. Each student takes at least one math class and exactly five of the 40 students take only one math class. How many students take all three mathematics classes?

A. 5 B. 6 C. 8 D. 9 E. 10

20. A professor has 3 sections of statistics classes and the section sizes are 20, 30, and 40 respectively. The average of an exam is 70 for the first section and 72 for the second section. The average of all his sections is 70. What is the average of his third section?

A. 68.5 B. 69 C. 69.5 D. 70 E. 71

21. Given that $a = \cos\left(\frac{\pi}{8}\right)$, express the perimeter of a regular octagon inscribed in a circle of radius 4 in terms of $a$.

A. $64\sqrt{1-a^2}$ B. $24a$ C. $\frac{16\pi}{a}$ D. $64\sqrt{2}$ E. $32(1-a^2)$

22. In how many ways can a group of 12 tennis players be divided into four equal groups?

A. 165 B. 495 C. 9075 D. 14520 E. 15400

23. In how many ways can a set of two different positive integers less than 100 be chosen?

A. 4851 B. 4950 C. 5050 D. 9801 E. 10000

24. Thirteen people on a softball team show up for a game. Of the 13 people who show up, 3 are women. How many ways are there to choose 10 players to take the field if at least one of these players must be a woman?

A. 175 B. 285 C. 505 D. 3718 E. 11154

25. The personal identification numbers (PINs) for automatic teller machines usually consist of four digits. You notice that most of your PINs have at least 0, and you wonder if the issuers use lots of 0s to make the numbers easy to remember. Suppose that PINs are assigned at random, so that all four-digit PINs are equally likely. What is the probability that a PIN assigned at random has at least one 0?

A. 0.2223 B. 0.3439 C. 0.4339 D. 0.4395 E. 0.6561

26. John ties his pet pit bull with a rope to one corner of a rectangular shed 12 ft long and 10 ft wide. The rope is 15 ft long. Find the approximate area outside the shed, measured in square feet, within the biting distance.

A. $26\pi$ B. $42\pi$ C. $169\pi$ D. $177\pi$ E. None of these

27. The UNCW sports website shop offers jerseys, bobblehead sets, and basketballs. One jersey, two bobblehead sets, and one basketball cost \$302. Two jerseys, one bobblehead set, and three basketballs cost \$284. Buying one of each item costs \$182. What is the price for each bobblehead set?

A. \$22 B. \$40 C. \$50 D. \$100 E. \$120

28. Determine the sum of all solutions of the following equation:

$$(1-6x)^2 - 4|1-6x| - 45 = 0.$$

A. $\frac{1}{3}$ B. $\frac{2}{3}$ C. 0 D. $-\frac{1}{3}$ E. $-\frac{2}{3}$

29. Find the coefficient of $x^5y^8$ in $(x+y)^{13}$.

A. 715 B. 792 C. 1287 D. 2002 E. 1716

30. Which of the following is equal to $\sin^{\frac{1}{2}} x \cos x - \sin^{\frac{5}{2}} x \cos x$?

A. $\sec^5 x \tan^3 x$
B. $\cos^3 x \sqrt{\sin x}$
C. $\cos^2 x \sin^{\frac{1}{2}} x$
D. $\cos^3 x \sin^2 x$
E. None of these