UNCW High School Math Contest Spring 2009

1. A spider has one sock and one shoe for each of its eight legs. Assuming that the sock for each leg must be put on before the shoe, in how many different orders can the spider put on its socks and shoes?

   A. $8!$ *B.* $2^8 8!$ *C.* $(8!)^2$ *D.* $\frac{16!}{2^8}$ *E.* $16!$

2. If m and n are integers such that $2m - n = 3$, what are the possible values of $m - 2n$?

   A. -3 only *B.* 0 only *C.* multiples of 3 *D.* any integer *E.* none of these

3. Five counterfeit coins are mixed with nine authentic coins. If two coins are drawn at random, what is the probability that one coin is authentic and one is counterfeit?

   A. $\frac{4}{9}$ *B.* $\frac{45}{91}$ *C.* $\frac{45}{182}$ *D.* $\frac{45}{196}$ *E.* $\frac{5}{9}$

4. In the number 7^{1000}, what is the digit in the units place?

   A. 7 *B.* 3 *C.* 0 *D.* 9 *E.* 1

5. The sum of three numbers is 98. The ratio of the first to the second is 2/3; the ratio of the second to the third is 5/8. What is the second number?

   A. 30 *B.* 73 *C.* 73.5 *D.* 20 *E.* 171.5

6. Find the sum of all solutions for the given equation,

$$x + \frac{1}{1 + \frac{1}{x}} = b, \quad \text{where } b > 0,$$

   A. $2 - b$ *B.* $b - 2$ *C.* $b + 2$ *D.* $2b$ *E.* $(b - 2)/2$

7. The volume of the pyramid with vertices at $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$, $(0, 0, 0)$ is:

   A. $\sqrt{3}$ *B.* 1/3 *C.* 1/4 *D.* 1/6 *E.* 2/3

8. The first two terms of a geometric sequence are 16 and 12, the fifth term is:

   A. 81/16 *B.* 6 *C.* 0 *D.* 243/64 *E.* None of those

9. Given that $f(x) = x^3 - 2$ and $g(x) = 5 - x - x^2$, $f^{-1}(g(-3))$ equals:

   A. 1 *B.* -3 *C.* -1 *D.* $-1/3$ *E.* 1/3

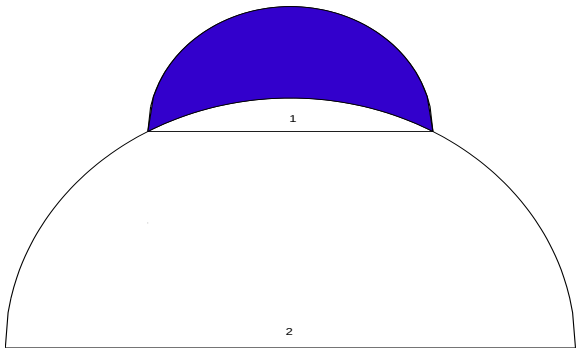
10. Given $8\sin^2\theta = 5 + 10\cos\theta$, determine the value of $\cos\theta$.

    A. $\sqrt{2}/2$ *B.* 1/2 *C.* 1/3 *D.* 1/4 *E.* 2/3

11. The unit's digit (one's digit) of the following sum is:

$$0! + 1! + 2! + 3! + \cdots + 50!$$

    A. 14 *B.* 13 *C.* 4 *D.* 3 *E.* None of those



12. The area of the shaded region in the above figure is

    A. $(6\sqrt{3}-\pi)/24$ *B.* $(3\sqrt{3}+\pi)/12$ *C.* $\pi/12$ *D.* $3\pi/4$ *E.* None of those

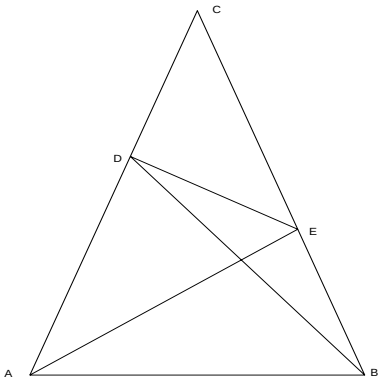
13. At a party where 14 couples were present, if each person shook hands with every other person except their spouse, how many handshakes took place?

    A. 162 *B.* 364 *C.* 378 *D.* 91 *E.* 351

14. Find the perimeter of a right triangle whose area is 40 cm^2 and whose hypotenuse is 15 cm.

    A. $15+5\sqrt{7}$ *B.* $15+\sqrt{265}$ *C.* $15+\sqrt{305}$ *D.* $15+\sqrt{385}$ *E.* $15\sqrt{545}$

15. Triangle ABC is isosceles with $CA = CB$. The angles $\angle ABD$, $\angle BAE$ and $\angle C$ have measures 50^o, 40^o and 40^o respectively. Find the measure of $\angle AED$.

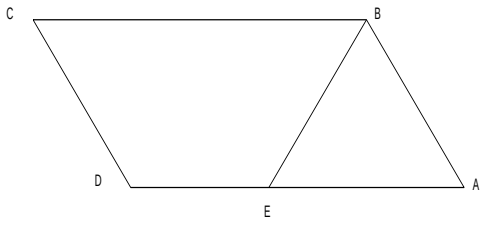


    A. 30^o *B.* 35^o *C.* 40^o *D.* 45^o *E.* 50^o

16. A given circular cylindrical can is made up of a square piece of metal and two circular disks each with diameter c inches. What is the volume of this cylindrical can?

    A. $\pi^2c^3/2$ *B.* $\pi^2c^3/4$ *C.* $2\pi^2c^3$ *D.* $\pi^2c^2/5$ *E.* $2\pi^2c^3/3$

17. Find the area of parallelogram $ABCD$, given that $AB = BE = ED = 2$, and $\angle ABE = 90^o$.



A. $1 + 2\sqrt{2}$ *B.* $2 + 2\sqrt{2}$ *C.* $4 + 2\sqrt{2}$ *D.* $4 + \sqrt{2}$ *E.* $2 + \sqrt{2}/2$

18. A triangle with sides 6, 8, and 10 has its shortest side doubled in length while the other two sides remain the same. What is the area of the new triangle?

A. $5\sqrt{63}$ *B.* $10\sqrt{55}$ *C.* 67 *D.* $2\sqrt{75}$ *E.* 35

19. If $\log_3(ab)^2 + \log_3(bc)^2 + \log_3(ac)^2 = 20$ for positive a, b, and c , what is the value of abc ?

A. 81 *B.* $20^{3/4}$ *C.* 729 *D.* $20^{3/2}$ *E.* 243

20. What is the sum of the digits of the integer solution to $4+\sqrt{11 + \sqrt[3]{-10 + \sqrt[5]{3x - 4}}} = 7$?

A. 5 *B.* 4 *C.* 3 *D.* 2 *E.* 6

21. Using the assumption that the probability that a baby is a boy is $\frac{1}{2}$ and the probability that a baby is a girl is $\frac{1}{2}$, determine the probability that a family with four children has two boys and two girls.

A. $\frac{2}{3}$ *B.* $\frac{2}{5}$ *C.* $\frac{1}{3}$ *D.* $\frac{1}{2}$ *E.* $\frac{3}{8}$

22. Rewrite the expression in terms of u and v, assuming u and v are positive.

$$\sec(\sin^{-1}(u) + \tan^{-1}(v))$$

A. $\sqrt{1+u^2}/(\sqrt{1-v^2} - uv)$ *B.* $\sqrt{1+v^2}/(\sqrt{1-u^2} - uv)$
C. $\sqrt{1+v^2}/(\sqrt{1-u^2} + uv)$ *D.* $\sqrt{1+u^2}/(\sqrt{1-v^2} + uv)$
E. $(\sqrt{1-v^2} - uv)/\sqrt{1+v^2}$

23. Find the exact value of $\cos 1^o \cdot \cos 2^o \cdot \cos 3^o \cdot \cdots \cdot \cos 45^o \cdot \csc 46^o \cdot \cdots \cdot \csc 89^o$.

A. $1/2$ *B.* $2/3$ *C.* $\sqrt{2}/2$ *D.* $\sqrt{2}$ *E.* $2\sqrt{2}$

24. Find the equation of the line containing the centers of two circles

$$x^2 + y^2 - 4x + 6y + 4 = 0 \qquad x^2 + y^2 + 6x + 4y + 9 = 0$$

A. $x + 5y = -13$ *B.* $x - 3y = -12$ *C.* $2x - 2y = 10$ *D.* $x - 5y = 12$
E. $5x - y = 13$

25. Find the coefficient of x^5 in the expansion of $(3x-2)^{13}$.

A. 81161696 *B.* 82261696 *C.* 80061696 *D.* 80082696 *E.* 80061608

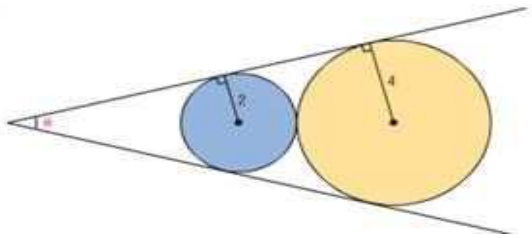
26. You teach 40 students. Blue is a favorite color of 22 students; 19 own cars; 13 like flowers; blue is the favorite color of 10 students who own cars; blue is the favorite color of 7 students who like flowers; 8 own cars and like flowers; and blue is the favorite color of 5 students who own cars and like flowers. How many students do not like flowers, do not like the color blue, and do not own a car?

A. 12 *B.* 4 *C.* 6 *D.* 20 *E.* 8

27. A bicyclist travels 2 miles at 5 minutes per mile, 4 miles at 4 minutes per mile and 6 miles at 2 minutes per mile. What is his average speed as minutes per mile for the entire trip, round your answer to one decimal digit.

A. 3.8 *B.* 3.1 *C.* 3.2 *D.* 3.0 *E.* 4.0

28. Find the value of the angle (see the figure) in degrees rounded to the nearest tenth of a degree.



A. 37.8^o *B.* 38.9^o *C.* 39.2^o *D.* 39.5^o *E.* 40.3^o

29. An unusual die has the numbers 2, 2, 3, 3, 5, and 8 on its six faces. Two of these dice are rolled, and the two numbers on the top faces are added. How many different sums are possible?

A. 18 *B.* 10 *C.* 8 *D.* 6 *E.* 9

30. If $x = \tan\theta$, express $\cos(2\theta)$ as a function of x.

A. $(1+x^2)/(1-x^2)$ *B.* $(1-x^2)/(1+x^2)$ *C.* $1/\sqrt{1-x^2}$

D. $(1-x)/(1+x)$ *E.* $(1-x)^2/(1+x^2)$