High School Mathematics Contest **Spring 2007**

1. Solve the equation $\log_2\left(2\sqrt{17-2x}\right) = 1 - \log_{\frac{1}{2}}(x-1)$.

   A. 2 B. 2.5 C. 3 D. 3.5 E. 4

2. Given that a,b are the solutions of the equation $x^2 + mx - m = 0$, find $a^3 + b^3$.

   A. $m + m^2$ B. $-m^2(m+3)$ C. $2m+1$ D. m^3 E. $m^2 + 4m + 1$

3. Maximize $z = xy$ subject to $5x + 7y = 20$.

   A. 1 B. $\frac{20}{7}$ C. $\frac{1}{7}$ D. 5 E. 7

4. How many real solutions does the equation $|2 - |1 - |x|| = 1$ have?

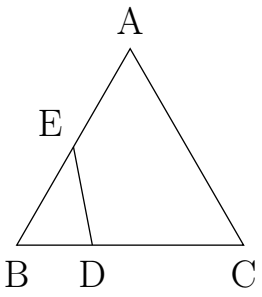
   A. 1 B. 2 C. 3 D. 4 E. 5

5. Find the sum of the solutions of the equation

$$\log 3 + \log\left(9^{x-3} + 16\right) = 1 + \log\left(3^{x-3} + 4\right).$$

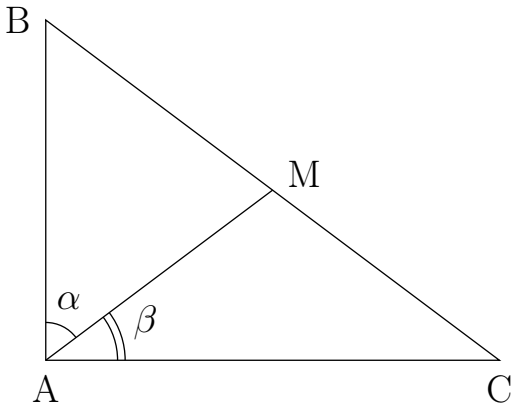
   A. $3 + \log 2$ B. $3 + \log_3 2$ C. $5 + 3\log_3 2$ D. $3 - \log_3 2$ E. $2 - \log 2$

6. Triangle ABC is equilateral with $|AB| = a$. Find $|CE|$ if $|BD| = \frac{a}{3}$, and $|AE| = |ED|$.



A. a B. $\frac{2a}{3}$ C. $\frac{3a}{4}$ D. $\frac{13a}{15}$ E. $\frac{a}{2}$

7. Right triangle ABC with $m(A) = 90^\circ$ has M as the midpoint of $|BC|$. If $\alpha = 2\beta$, then $\frac{|AB|}{|AC|}$ equals



A. 1 B. $\sqrt{3}$ C. $\sqrt{2}$ D. $\frac{\sqrt{3}}{3}$ E. $\frac{1}{2}$

8. The expression $2(1+\cos x)-\sin^2 x$ is the same as:

A. $(1-\cos x)^2$ B. $1-\cos^2 x$ C. $(1+\cos x)^2$ D. $1+\cos^2 x$ E. $(1+\sin x)^2$

9. The expression $\binom{n}{k}+\binom{n}{k+1}$ (where $n \geq k \geq 1$) is the same as:

A. $\binom{n}{k+1}$ B. $\binom{n+1}{k}$ C. $\binom{n+1}{k-1}$ D. $\binom{n}{k+1}+\binom{n+1}{k}$ E. $\binom{n+1}{k+1}$

10. Which of the following divides n^3+11n for all positive integers n?

A. 5 B. 7 C. 6 D. 4 E. 9

11. Simplify the expression $\left(2e^{5x+3}\right)^2+4e^{10x+6}+e^{10x+7}$.

A. $7e^{10x+6}$ B. $(8+e)e^{10x+6}$ C. $6e^{10x+6}+e^{10x+7}$ D. $8e^{10x+7}$ E. $9e^{10x+7}$

12. What is the coefficient of x^4y^6 in the expansion of $(2x+y)^{10}$?

A. 3360 B. 210 C. 16 D. 24 E. 13440

13. A color-blind individual has only 16 pairs of socks, 10 identical black pairs and 6 identical navy blue pairs. He never matches his socks after washing and just throws them in his sock drawer. Assuming that all 32 socks in the drawer are unmatched, what is the probability that if he randomly selects two socks, they will match?

A. $\frac{95}{248}$ B. $\frac{33}{248}$ C. $\frac{16}{31}$ D. $\frac{1}{2}$ E. $\frac{245}{496}$

14. A specially made die has one face with one dot, two faces with two dots, and three faces with three dots. If this die is rolled 20 times, approximately how many rolls would you expect to see with two dots facing up?

A. 5 B. 10 C. 13 D. 7 E. 11

15. A company randomly generates a 4-digit security code for all its employees. However, security codes containing the same four digits are not allowed (i.e., 0000, 1111, ... are not allowed). How many different security codes can be issued?

A. 9990 B. 5030 C. 6561 D. 24 E. 30

16. Let ℓ be equal to the length of a string, in feet, needed to wrap around the equator of the earth. What is the minimum amount (in feet) of additional string needed to have the string at distance 1 ft above the equator? Assume that the earth is a perfect sphere.

A. 1 ft B. 7 ft C. 1000 ft D. 4000 miles E. not enough information

17. If John runs around a 1 mile track in 20 minutes, how fast does John have to go around the second time to average 6 miles per hour (mph)?

A. 7 mph B. 8 mph C. 9 mph D. 10 mph E. can't go fast enough

18. For $-1 < x < 1$, $\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + \cdots$. Find $3\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^3 + 5\left(\frac{1}{2}\right)^4 + \cdots$.

A. 1 B. 2 C. 4 D. 16 E. ∞

19. Determine the value of $x + y$ given that $2^x = 4^{a+1}$ and $3^y = 9^{-a+3}$.

A. 1 B. 8 C. $\ln(-a + 13)$ D. 52 E. not enough information

20. Find $[f(x)]^2 - [g(x)]^2$ given that $f(x) = e^x + e^{-x}$ and $g(x) = e^x - e^{-x}$.

A. $2e^{-2x} + 2e^{2x}$ B. 4 C. $2e^{2x}$ D. 0 E. $2e^{-2x}$

21. Let $\triangle ABC$ be equilateral and let D and E be the midpoints of the sides AB and AC respectively. Find the ratio of the area of $\triangle ABC$ to $\triangle ADE$.

A. $\frac{1}{3}$ B. $\frac{1}{4}$ C. $\frac{1}{2}$ D. 3 E. 4

22. How many 5-digit numbers are there for which the sum of the digits is 7?

A. 28 B. 64 C. 190 D. 210 E. 330

23. How many digits are there in the integer value 2^{100} after it has been multiplied out?

A. 30 B. 31 C. 32 D. 33 E. 34

24. Determine the number of real solutions of the following system.

$$\begin{cases} x + y = 2 \\ xy - z^2 = 1 \end{cases}$$

A. 2 B. 3 C. 4 D. 5 E. none of these

25. Determine which of the following numbers **cannot** be represented in the form $65x^2 + 894xy + 3074y^2$ for some integers x, y.

A. 221 B. 493 C. 1813 D. 3103 E. 3509

26. Given that $n = 62773913$ is relatively prime to 62758068 numbers in the range $1, 2, 3, \ldots, 62773912$, find the sum of the factors of n.

A. 12645 B. 12534 C. 14166 D. 14647 E. 15846

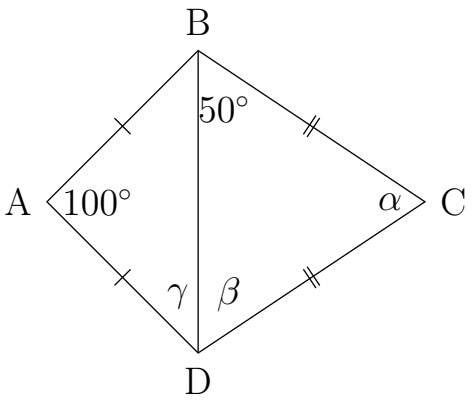
27. Determine the number of odd entries in the 2007th row of Pascal's triangle.

A. 64 B. 256 C. 435 D. 512 E. 1025

28. Halley's comet appeared most recently in the years 1835, 1910 and 1986. The next appearance will be in 2061. Find the smallest prime factor of $1835^{1910} + 1986^{2061}$.

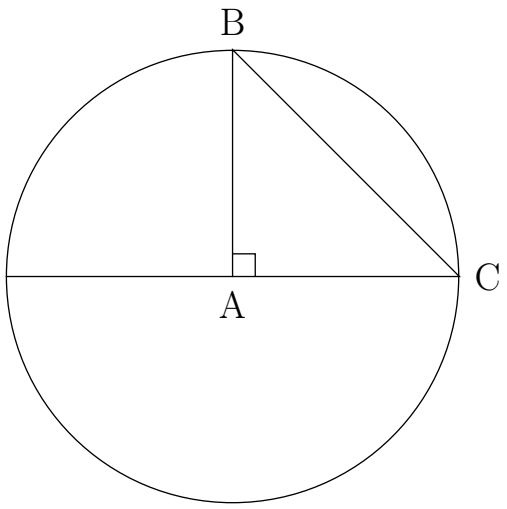
A. 3 B. 5 C. 7 D. 23 E. 31

29. Find the sum of the angles α,β,γ.



A. 106° B. 142° C. 160° D. 170° E. 172°

30. Right triangle ABC is inscribed in the circle below. If the circle has area 25π, find the length of the chord BC.



A. $\sqrt{25}$ B. $\sqrt{50}$ C. $\sqrt{100}$ D. $\sqrt{10}$ E. $6\sqrt{2}$

UNCW HIGH SCHOOL MATH CONTEST

Department of Mathematics and Statistics

March 26, 2007

Scoring
Each question answered correctly scores 4 points.
Each question left blank scores 1 point.
Each question answered incorrectly scores 0 points.